



# Topic 4: Stability and fault analysis tools for Smart Grids

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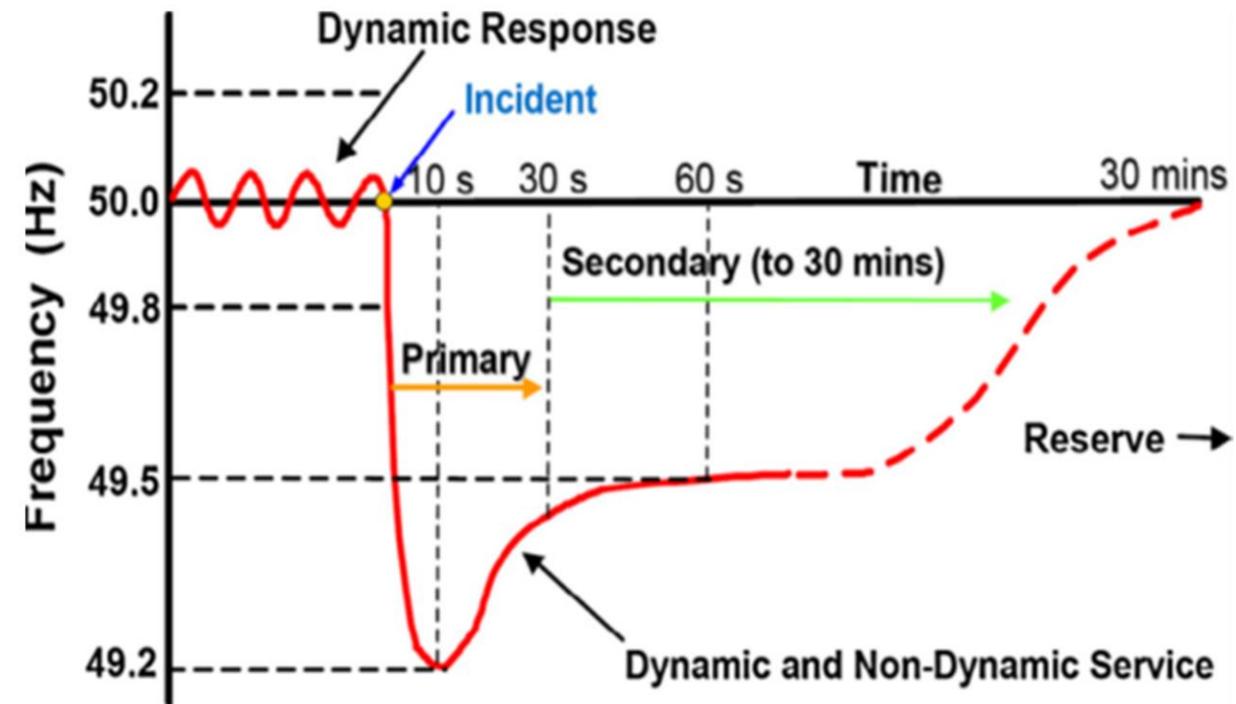
## 4.1. Power Grid operations control

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- The main functions of operations control are satisfying the instantaneous load on a second - to - second and minute - to - minute basis.
- Some of these functions are:
  1. Load frequency control (LFC)
  2. Automatic - generation control (AGC)
  3. Network topology determination (NTD)
  4. State estimation (SE)
  5. On - line load flow and contingency studies
  6. Schedule of transactions (ST)
  7. Economic dispatch calculation (EDC)
  8. Operating reserve calculation (ORC)
  9. Load management system (LMS)

## 4.1. Power Grid operations control

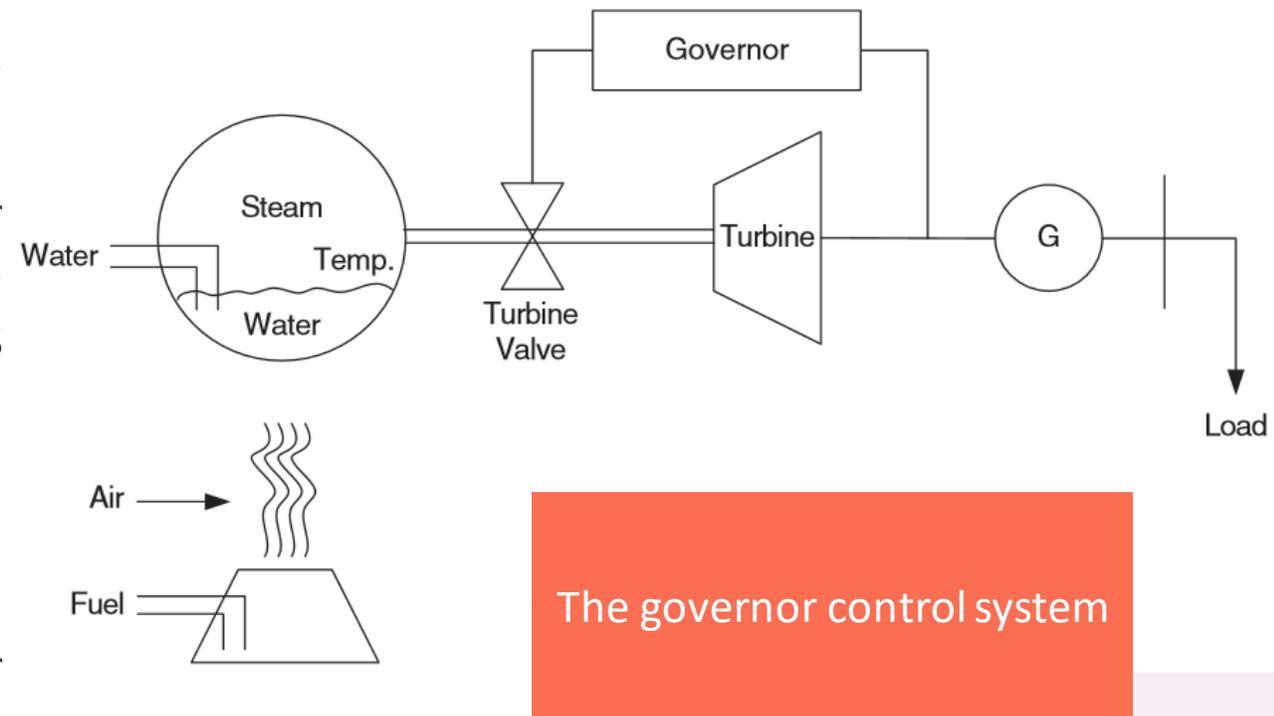
- The decision time of operations control is from dynamic response in a fraction of a cycle in LFC, to seconds for automatic - generation control, to 5 – 10 minutes for economic - dispatch calculations, and from a second up to 30 minutes for a load management system.
- However, with the implementation of a smart grid system with a high penetration of renewable green energy sources and a smart metering system, we will have a more - complex power system.



The LFC graph

## 4.2. Load frequency control (LFC)

- LFC is also referred to as the governor response control loop as shown in the next Figure.
- As the load demand of the power system increases, the speed of the generators decreases and this reduces the system frequency.
- Similarly, as the system load-demand decreases, the speed of the system generators increases and this increases the system frequency. The power system-frequency control must be maintained for the power grid to remain stable.



## 4.2. Load frequency control

- In the AC power grids, all generating sources are operating in parallel and all (inject) supply power to the power grid. This means that all power sources must be operating at the same system frequency. The system operating frequency in the United States is 60Hz and at 50Hz in the rest of the world. The generators are operating at the system frequency; they are all synchronized and operating at the same synchronized speed: all are supplying (injecting) power to the power grid. The synchronized speed can be computed as

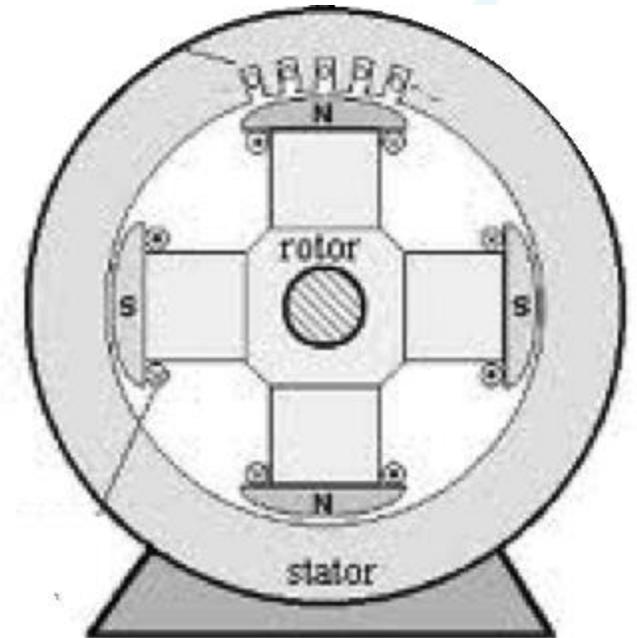
$$\omega_{syn} = \frac{2}{P} \omega_s$$

- where  $\omega_s = 2\pi f_s$  and  $f_s$  is the system frequency. In revolutions per minute (rpm), we have:

$$n_{syn} = \frac{120f_s}{P} \text{ rev / min - rpm}$$

## 4.2. Load frequency control

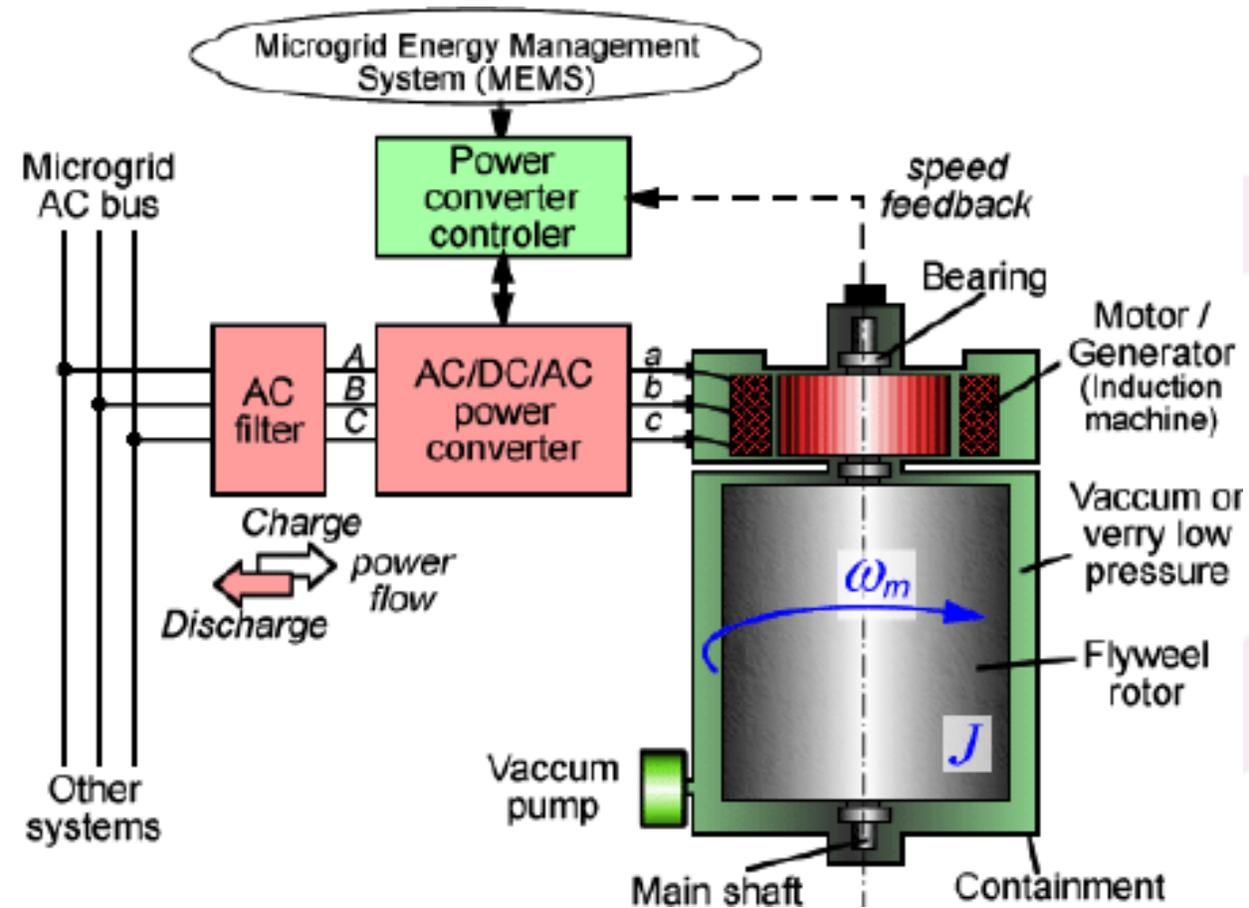
- In the previous equation,  $P$  is the number of poles and  $f_s$  is the generator frequency. Therefore, for a two - pole machine, operating at 60Hz ( $f = 60\text{Hz}$ ), the shaft of the machine is rotating at 3600 rpm. If the prime mover power has a slower speed, such as the hydropower unit, the generator has more poles. For example, if  $P = 12$ , the speed is 600 rpm and still the unit operates at 60 Hz.
- Synchronized operation means that all generators of the power grid are operating at the same frequency and all generating sources are operating in parallel. This also means that all generating units are operating at the system frequency regardless of the speed of each prime mover. In AC systems, the energy cannot be stored; it can only be exchanged between inductors and capacitors of the system and is consumed by loads.



4-pole generator

## 4.2. Load frequency control

- Therefore, for an AC system to operate at a stable frequency, the power generated by AC sources must be equal to the system loads.
- However, the loads on the system are controlled by the energy users, i.e., when we turn on/off lights, we increase/reduce the system load. In response to load changes, the energy is supplied from the inertia energy stored in the massive mass of a rotor.



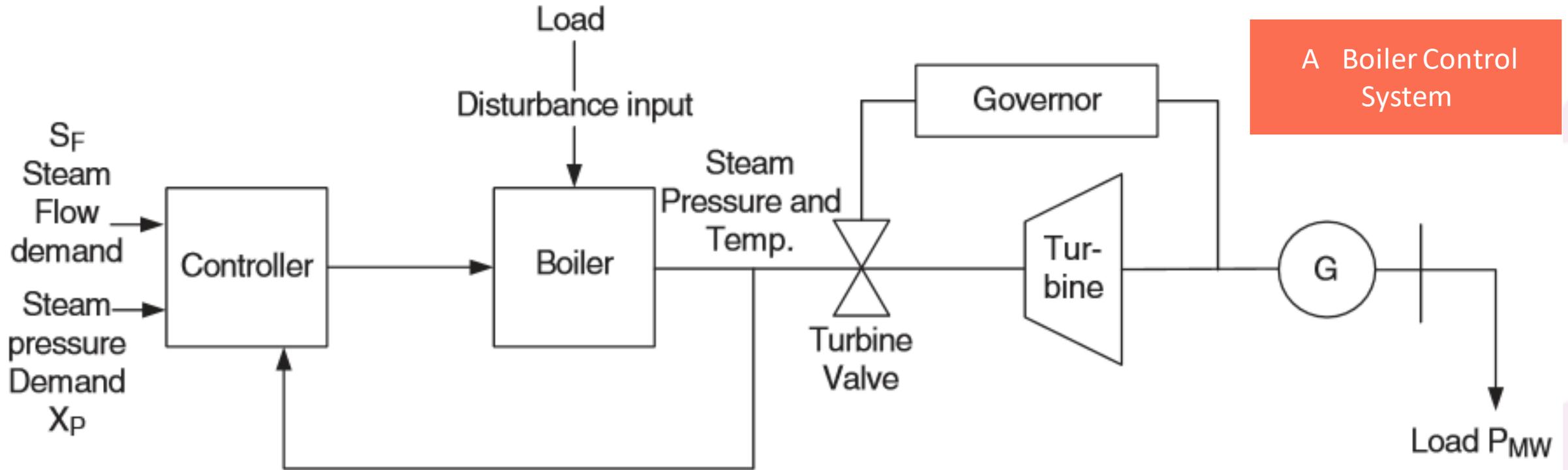
## 4.2. Load frequency control

- However, at every instant, the balance between energy supplied to the grid and the energy consumed by loads plus losses are maintained. This concept can be expressed as

$$\sum_{i=1}^{n_1} P_{G_i} = \sum_{i=1}^{n_2} P_{L_i} + P_{losses}$$

- where  $P_{G_i}$  is the power generated by generator  $i$ ,  $P_{L_i}$  is the power consumed by load  $i$  and the  $n_1$  is the number of the system generators and  $n_2$  is the number of the system loads. The transmission line losses are designated by  $P_{losses}$ .
- As can be expected, as the system load demand at time  $t$  increases, we should expect that the system frequency decreases because the power system at that instant has more loads than at the instant  $t - k$  where  $k$  is the time step. In fact, this is precisely what happens at first.
- However, the system has a feedback loop that is called the load - speed control and as the system frequency drops, i.e., the prime mover shaft speed decreases, the feedback loop increases the input power to match the generation to the system load. This is called ***governing system control or load frequency control***.

## 4.2. Load frequency control



## 4.2. Load frequency control

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- The governor opens the turbine valves to increase the input power that in turn speeds up the shaft of the generator. However, with an increase in the system loads, the additional power generated matches the system generation to the system load and the system will operate at the system synchronized speed. The governor control keeps the turbine shaft speed constant at the desired synchronized speed to generate power at a synchronized system frequency. To ensure the safety of the boiler and turbine, the boiler control system controls the condition of steam that is expressed by steam pressure and steam temperature.
- The boiler control system controls the turbine valve in the desired position such that the steam pressure and temperature are within their specified range. The governor control feedback controls the turbine shaft speed — as the system load changes; the governor feedback opens or closes the turbine valves. However, the opening and closing turbine valves are dependent on steam conditions. The turbine valves can be opened or closed as long as the boiler steam conditions are within the desired range.

## 4.2. Load frequency control

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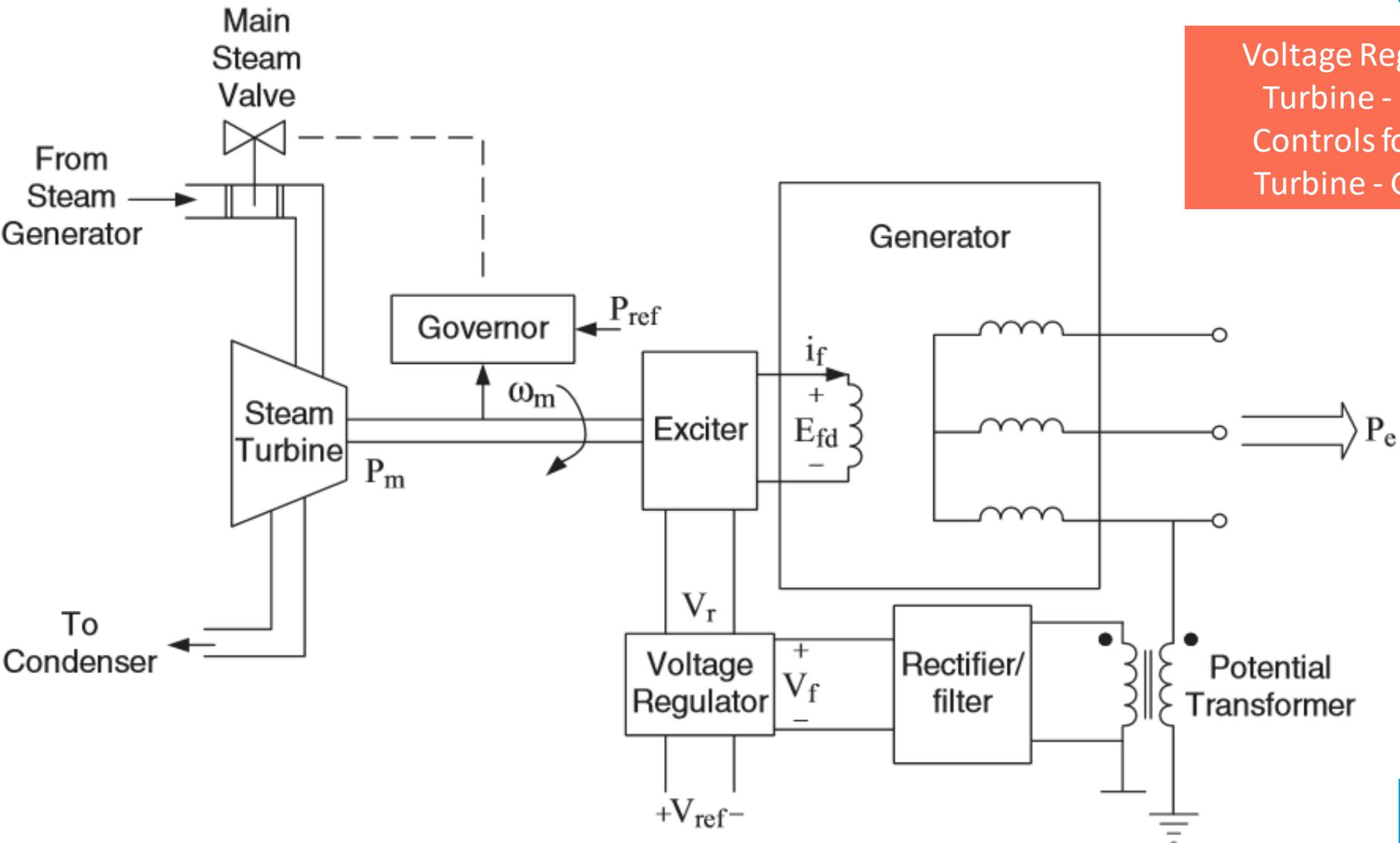
- To make sure the system generation will match the system load, two control methods are made. These methods are *turbine-following control and boiler follow-up control*.
- In the *turbine-following* control, the *turbine* generator is assigned the *responsibility of throttle pressure*. The turbine valves are controlled within a specified range that ensures that steam conditions, steam pressure, and temperature are within the safe range. The MW load demand corresponds to steam flow demand and it is controlled by the boiler. When the step increase in load control command is issued, the control command is sent to the boiler. The boiler control system then increases the fuel rate, feed water, and airflow, which increases the throttle pressure. The change in the throttle pressure is measured by the turbine control system. The turbine valves are controlled by the turbine control system. The turbine valves are opened to increase the steam flow and MW output of the generator. Note that when the steam flow increases due to the opening of the turbine valves, the turbine shaft accelerates.

## 4.2. Load frequency control

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- In *boiler follow* - up control, the *boiler* is assigned the *responsibility of throttle pressure*. The MW load demand is controlled by the turbine generator. In this mode of operation, a step increase in generation due to a step change in load demand goes directly to the turbine valves. The load demand increases, the turbine valves open, and hence the steam flow and MW output of the generator increases.
- However, the boiler is controlling the throttle pressure, and if the pressure drops out of the range assigned to the boiler, the boiler control system overrules the turbine control action to maintain the pressure. Both the proposed control systems can provide satisfactory control.
- The *boiler follow* - up control has a *faster response* and is widely used. The *turbine control* system has a slower response; however, it *protects the boiler* and ensures that steam is conditioned before energy is extracted from the boiler.

## 4.2. Load frequency control



Voltage Regulator and Turbine - Governor Controls for a Steam Turbine - Generator.

## 4.2. Load frequency control

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- By applying the mechanical power to the rotor winding that is supplied with DC current, a time-varying field is established in the air gap of the machine. Based on Faraday's law of induction, the voltage is induced on the stator windings.
- Again, because the generator is synchronized to the power grid, the power is injected into the system. A power generator is a three - terminal device. We set the field current of the generator to set the generator's terminal voltage:  $E = K \cdot I_f \cdot \omega$
- The open - circuit - induced voltage,  $E$  is a function of the machine dimensions that are given by the constant  $K$  and field current,  $I_f$  and shaft speed,  $\omega$ . By adjusting the field current, a generator can operate at leading or lagging power factor. We will study this concept later in this chapter. However, ***the reactive power,  $Q_G$  generated by machines must be equal to the total reactive loads and transmission lines' reactive losses:***

## 4.2. Load frequency control

$$\sum_{i=1}^{n_1} Q_{Gi} = \sum_{i=1}^{n_2} Q_{Li} + Q_{losses}$$

where  $Q_G$  is the reactive power generated,  $Q_L$  is the reactive power of load,  $Q_{losses}$  is the reactive power loss.

- We now introduce two important studies in a power system functioning:

1. **Power Flow Studies.** Given the schedule system generation, system load, and schedule system elements such as transmission lines and transformers, etc., we compute the system bus voltages and power flow on transmission lines. These conditions are expressed by the previous eqs. We often refer to bus voltages as system states that represent the voltage magnitude and phase angle at each bus. For power flow studies, we are interested in the system injection model: we do not include the generator impedance in the power grid injection model that describes the injected power at the terminal of the generator into the network model of the transmission system.

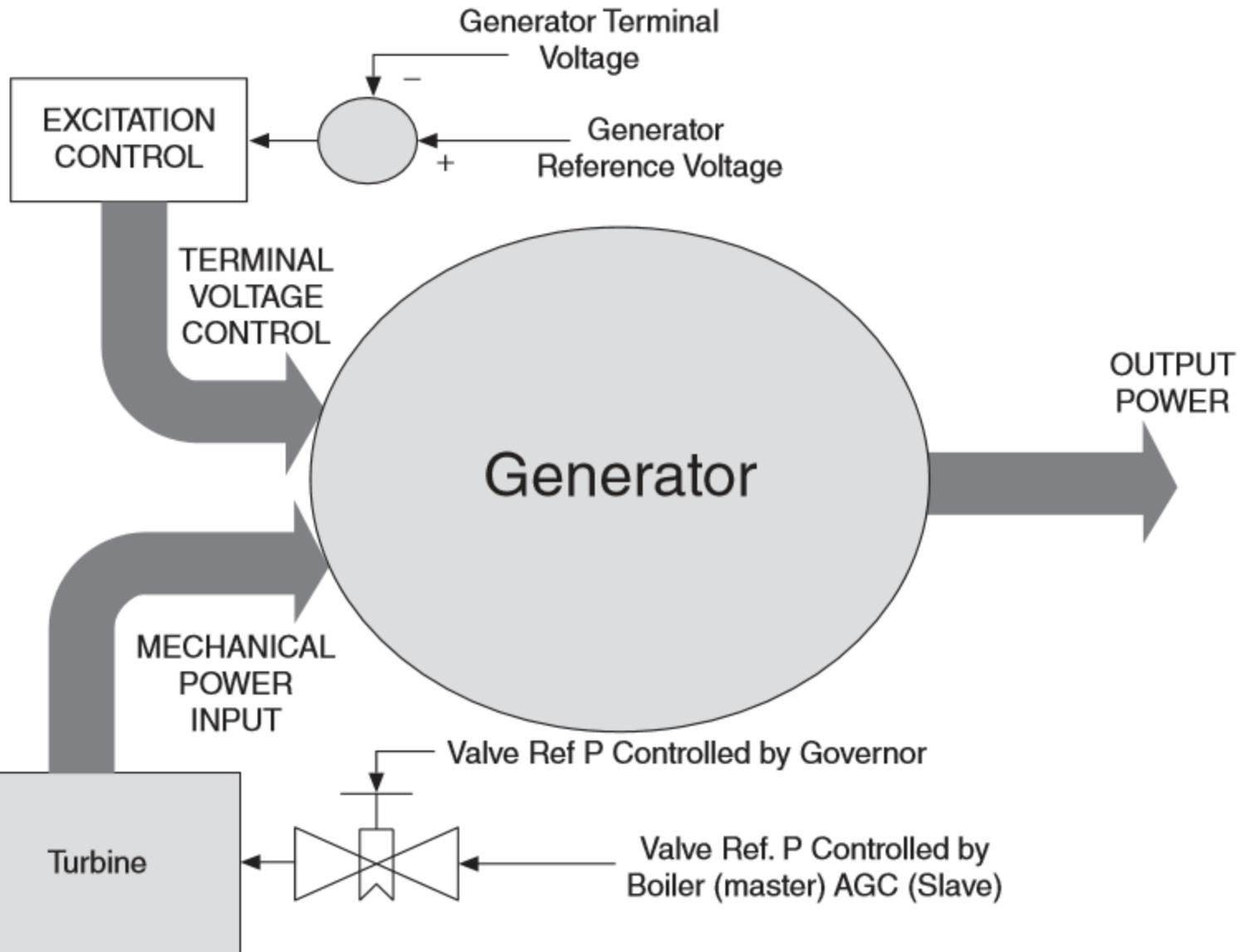
## 4.2. Load frequency control

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2. **Short-Circuit Studies.** Given the system model, the bus voltages, and load, we compute balanced and unbalanced *fault currents* that can flow on the system if a fault happens. Based on this study, we calculate the short-circuit currents that the breakers may experience upon occurrence of a fault. This study also provides the level of fault current throughout the system for setting relays of the protection system.

- In the short - circuit studies, the internal input impedance of generating sources must be included because they limit the fault current as it happens upon occurrence of a fault. Without internal input impedance of generating sources, the fault current would be infinite; this is unrealistic because the source would catch fire before an extremely high current is reached.

## 4.2. Load frequency control



A Generator as a Three - Terminal Device.

## 4.2. Load frequency control

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- Let us return to the operation of a generator: In the second terminal of Fig. shown before, we supply the mechanical power to the generator shaft, and in turn, we set up a time-varying flux in the air gap of the generator that will couple the windings located on the stator of the generator and produces the terminal voltage.
- The output power of the generator is injected into the power system network. The injected power and its power factor are controlled by controlling the field current and the terminal voltage.
- The dynamic range of a power system operation starts from start up— a transient condition to the steady - state operation. The dynamic duration of a power grid can be from a few cycles to several minutes. The generator excitation - control system can be subjected to dynamic perturbation from a few cycles to a few seconds as the field current of the generator is changed to a new voltage setting.

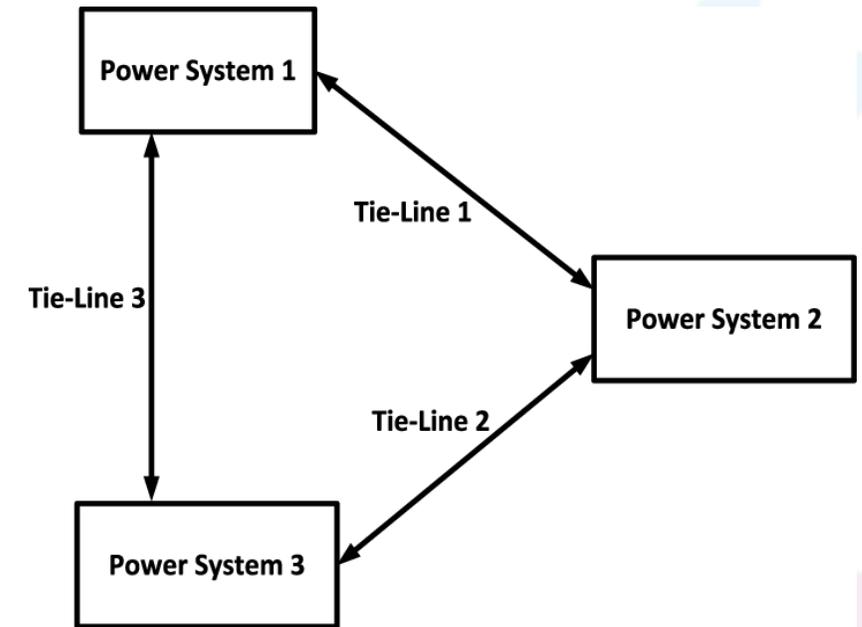
## 4.2. Load frequency control

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- When a power grid is subjected to an outage from the loss of a generator, the power grid will be subjected to the dynamic stability problem that can be stabilized if the power grid can provide the power needed to balance the system generation to the system load.
- For example, for a generator outage, the governors of all units within the power grid will react to a deficiency in needed power (that is a drop in the system frequency) and will inject additional power into the grid to match the generation to the system load. We can identify different dynamic problems that can affect a power grid:
  1. Electrical dynamics and excitation controls may have a duration of several cycles to a few seconds.
  2. Governing and LFC may have a dynamic duration of several seconds to a few minutes.
  3. A prime - mover and an energy supply control system may have a dynamic duration of several minutes. A prime-mover is a steam-generating power system.

## 4.3 Automatic Generation Control

- The system load has a general pattern of increasing slowly during the day and then decreasing at night. The cost of generated power is not the same for all generating units. Therefore, *more* power *generation* is assigned to the *least costly* units. In addition, a few lines connect one power grid to another neighboring power grid. These lines are referred to as tie lines.
- Tie lines are controlled to import or export power according to set agreed contracts. When power is exported from a power system to a neighboring power system through the *tie lines*, the exported power is considered as load; conversely when imported, it is considered as power generation.



## 4.3 Automatic Generation Control (AGG)

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- To control both the power flow through transmission tie lines and the system frequency, the concept of *area control error (ACE)* is defined as

$$ACE = \Delta P_{TL} - \beta \Delta f$$

where  $\Delta P_{TL} = P_{Sch} - P_{Actual}$   
 $\Delta f = f_s - f_{Actual}$

$P_{Sch}$ : The scheduled power flow between two power networks

$P_{Actual}$ : The actual power flow between two power networks

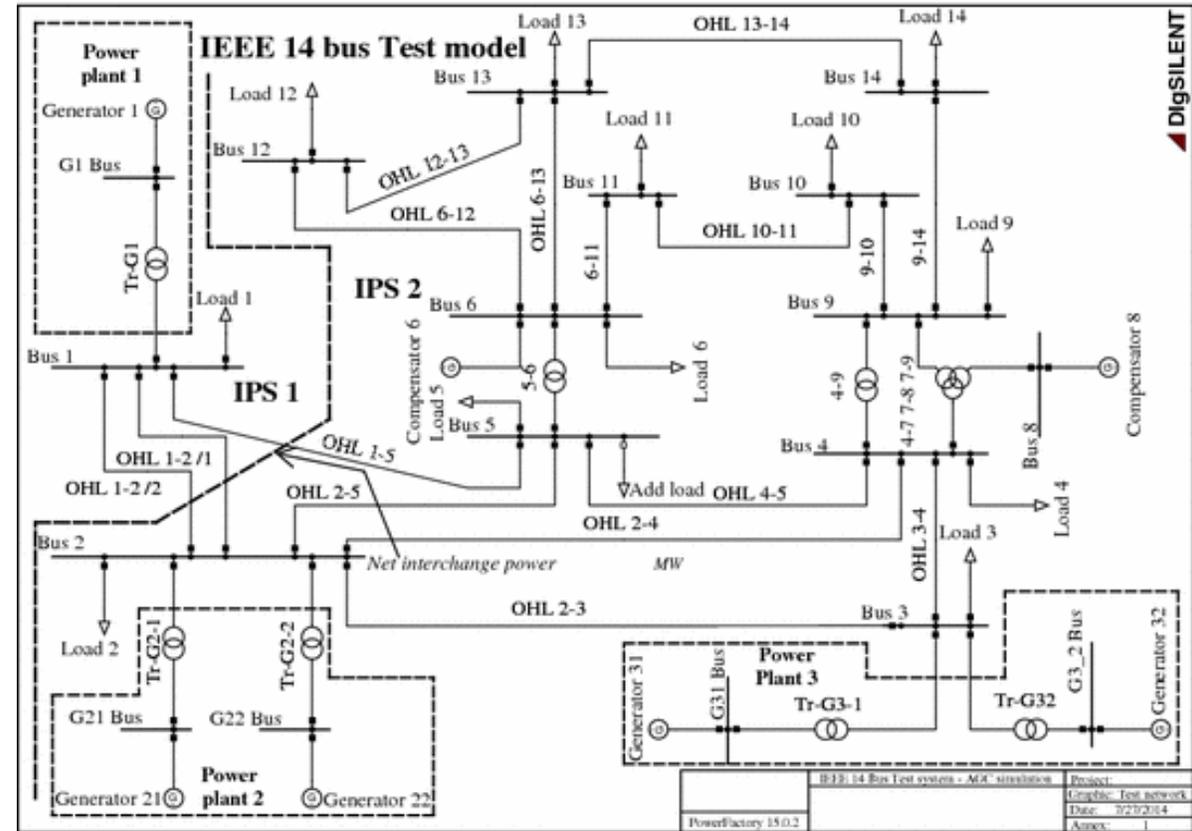
$f_s$ : The reference frequency, i.e., the rated frequency

$f_{actual}$ : The actual measured system frequency

$\beta$ : The frequency bias

## 4.3 Automatic Generation Control (AGG)

- The AGC (Automatic generation control) software control is designed to accomplish the following objectives:
  - Match area generation to area load, i.e., match the tie-line interchanges with the schedules and control the system frequency.
  - Distribute the changing loads among generators to minimize the operating costs.
- The above condition is also subject to additional constraints that might be introduced by power grid security considerations such as loss of a line or a generating station.



Modeling of AGC in Power systems

## 4.3 Automatic Generation Control

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- The first objective involves the supplementary controller and the concept of tie - line bias. The term  $\beta$  is defined as bias and it is a tuning factor that is set when AGC is implemented. A small change in the system load produces proportional changes in the system frequency. Hence, the area control error ( $ACE = \Delta_{PTL} - \beta \Delta f$ ) provides each area with approximate knowledge of the load change and directs the supplementary controller for the area to manipulate the turbine valves of the regulating units.
- To obtain a meaningful regulation (i.e., reducing the ACE to zero), the load demands of the system are sampled every few seconds. The second objective is met by sampling the load every few minutes (1 – 5 minutes) and allocating the changing load among different units to minimize the operating costs. This assumes the load demand remains constant during each period of economic dispatch.

## 4.3 Automatic Generation Control

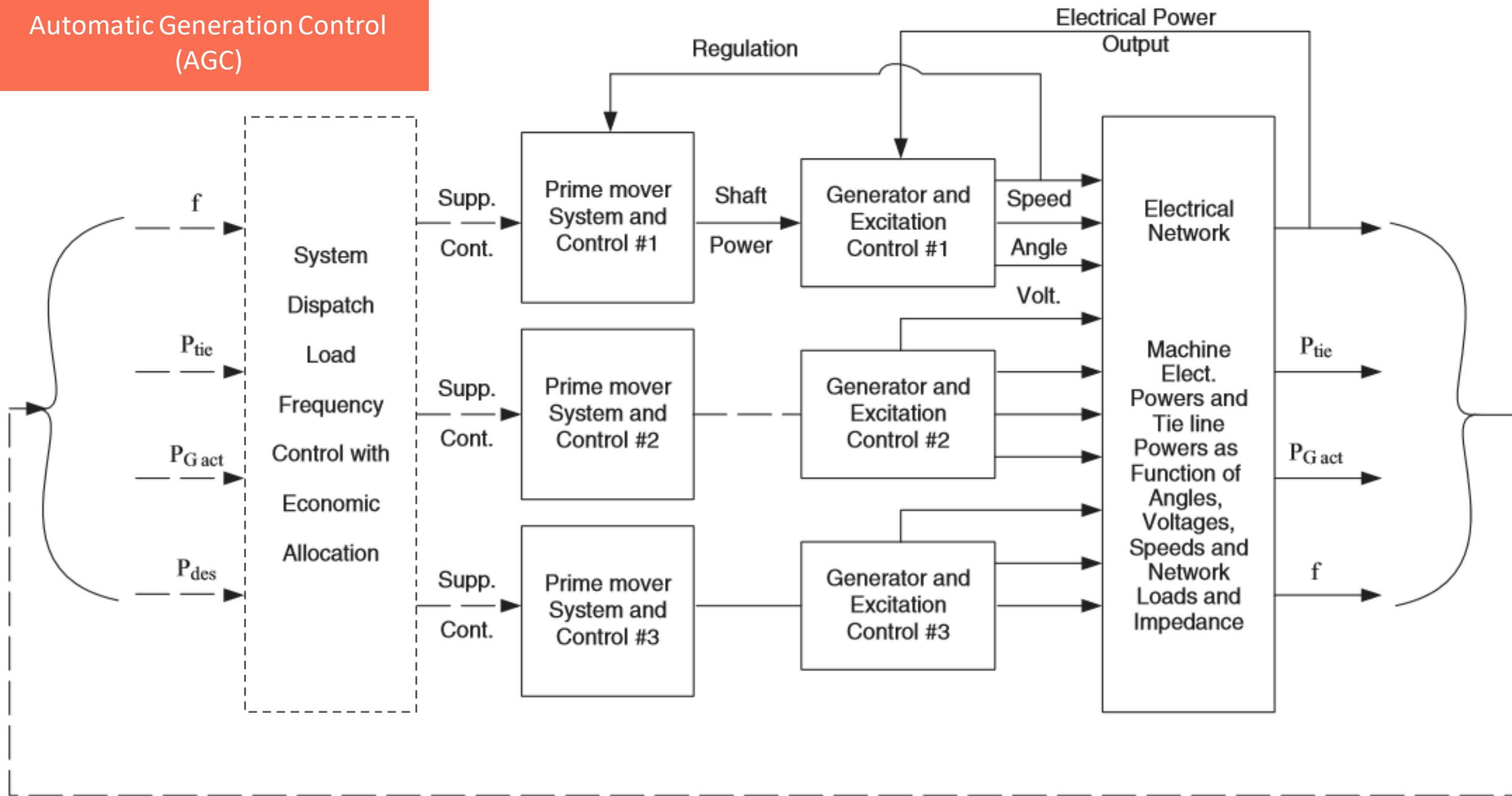
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- To implement the above objectives, nearly all AGC software is based on unit control. For unit  $i$ , the desired generation at time instant  $K$  is normally sampled every 2 or 4 seconds and is given by,

$$P_D^i(K) = P_E^i(K) + P_R^i(K) + P_{EA}^i(K)$$

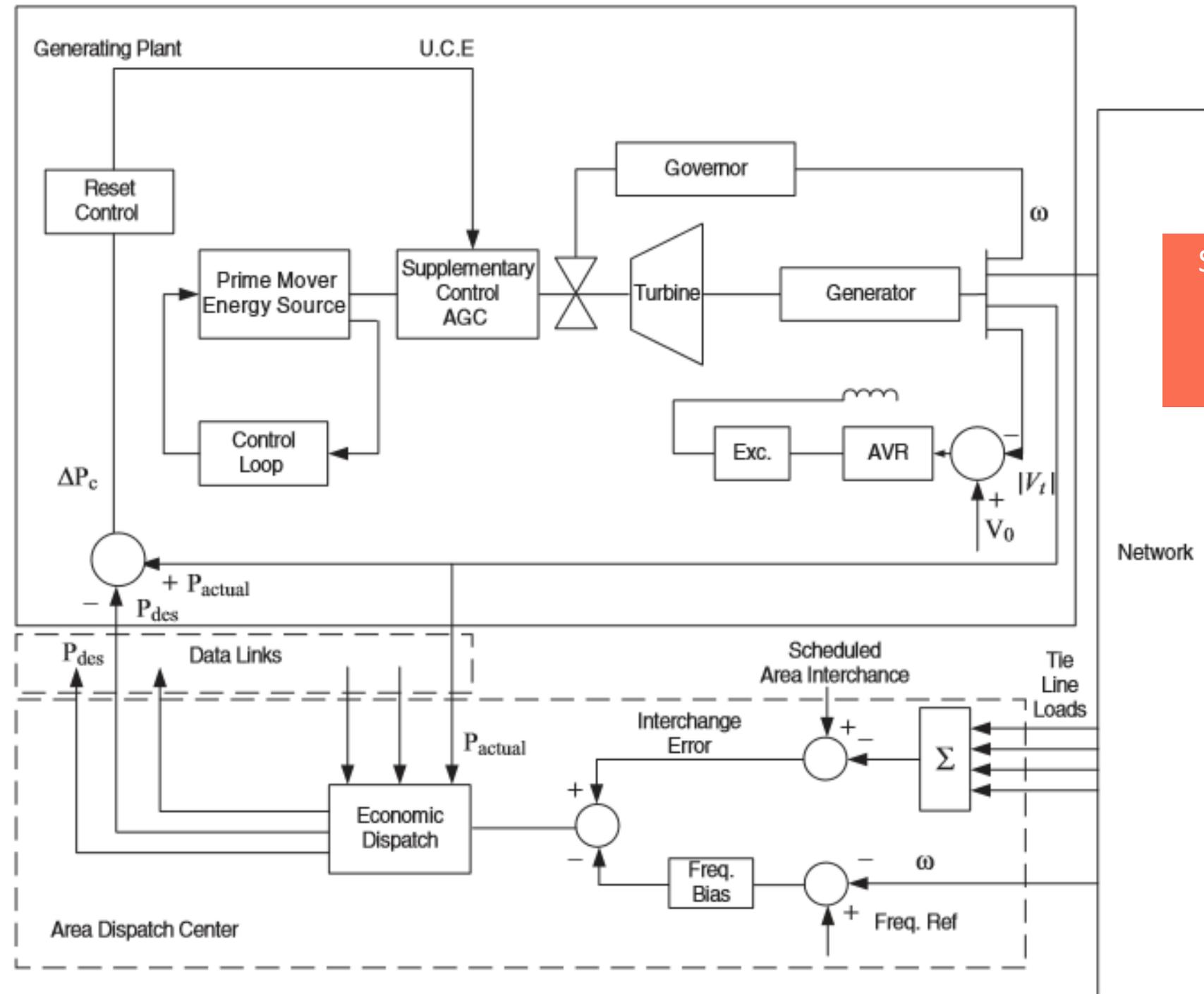
- where  $P_E^i$ ,  $P_R^i$  and  $P_{EA}^i$  are the *economic*, *regulating*, and *emergency assist* components of desired generation for unit  $i$  at time instant  $K$ , respectively.

# Automatic Generation Control (AGC)

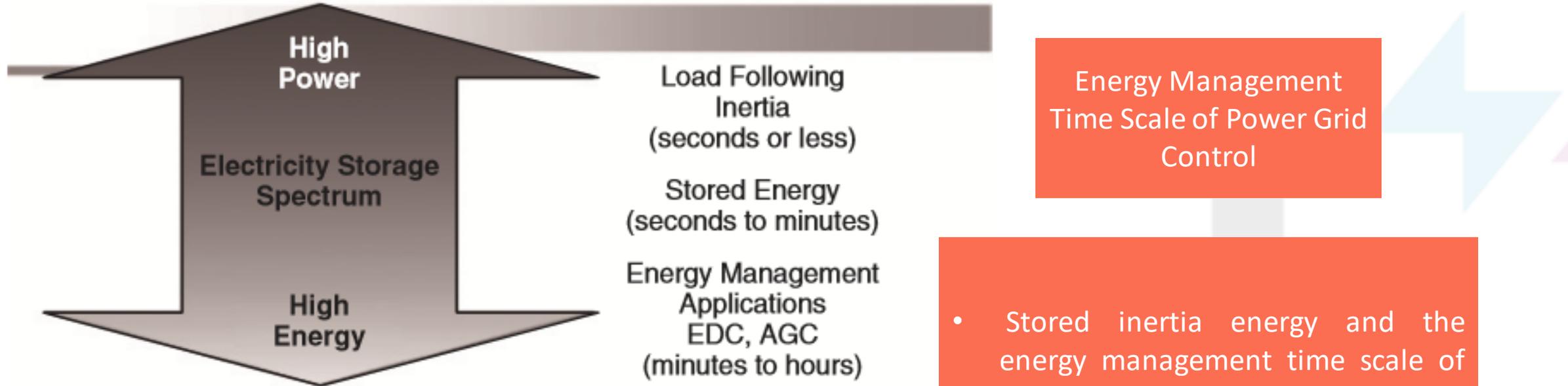


# Generation Control

Schematic Diagram of Load – Frequency Control System with Economic Dispatch



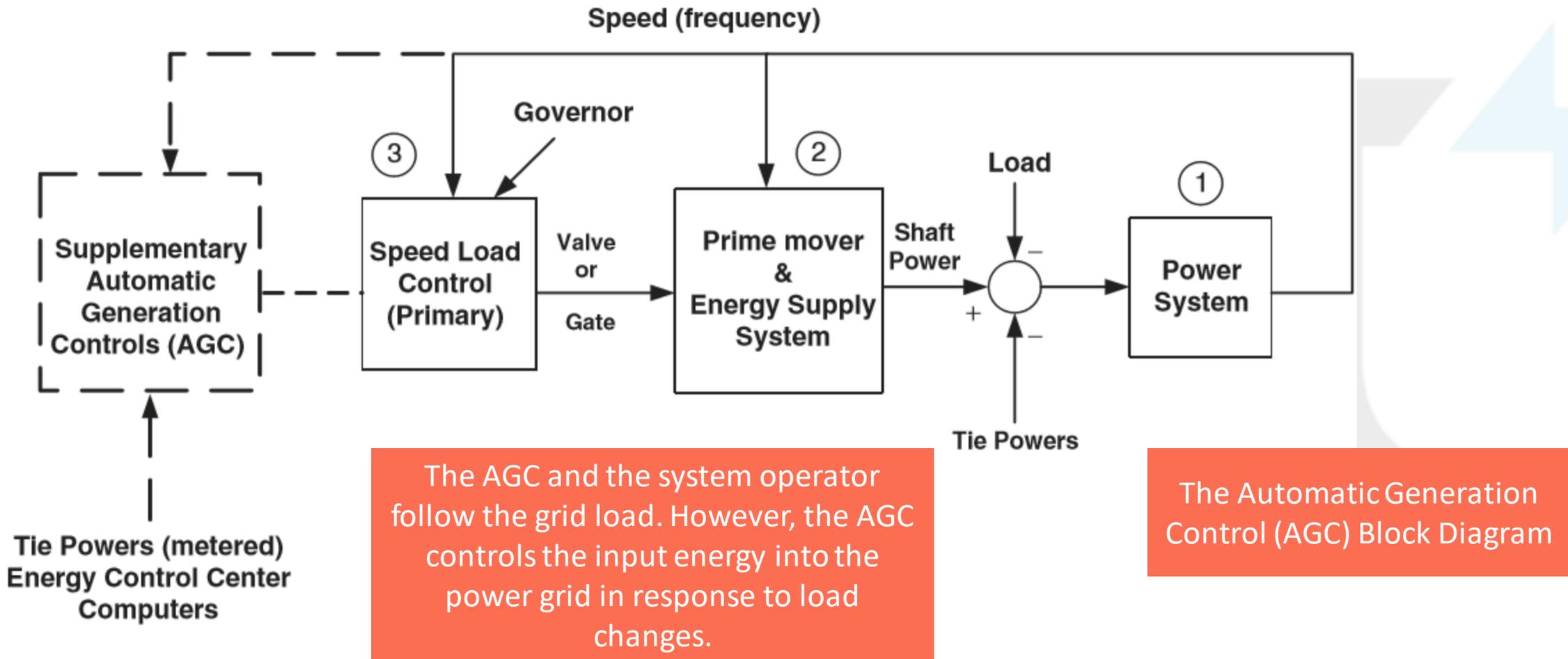
## 4.3 Automatic Generation Control



- Simply, when an energy user turns off a light, the load drop creates a high-frequency load fluctuation. Of course, when a large number of energy users turn their lights off, they create high- and low - frequency load fluctuations.

- Stored inertia energy and the energy management time scale of power grid control: The stored inertia energy in the rotor of generating units provides energy to the high - frequency load changes!

# 4.3 Automatic Generation Control



The AGC and the system operator follow the grid load. However, the AGC controls the input energy into the power grid in response to load changes.

The Automatic Generation Control (AGC) Block Diagram

## 4.3 Automatic Generation Control

- The AGC also controls the connected microgrids in a large interconnected power grid. The microgrid concept assumes a cluster of loads and its microsources, such as photovoltaic, wind, and combined heat and power (CHP) are operating as a single controllable power grid.
- To the local power grid, this cluster becomes a single dispatchable load. When a microgrid power grid is connected to a power grid, the microgrid bus voltage is controlled by the local power grid. Furthermore, the power grid frequency is controlled by the power grid operator. The *microgrid cannot change the power grid bus voltage* and the power grid *frequency*.



## 4.3 Automatic Generation Control

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- To understand why this is the case, we need to understand the control system that is used by power grid operators. Figure in Slide 29 depicts the prime mover, energy supply (steam or gas turbine), and governor (speed load control) system. These systems are located in the power - generating station.
- The supplementary controls and AGC are part of the EMS of the local power grid. The LFC system is designed to follow the system load fluctuation. As stated before, when the load changes, let's say as the load increases in the microgrids connected to the local power grid, then the inertia energy stored in the system supplies the deficiency in energy, to balance the load to generation. This energy is supplied by prime movers (stored energy in rotors).
- The balance between load and generation must be maintained for the local power grid to remain stable. When the balance between generation and load is disturbed, the dynamics of the generators and loads can cause the system frequency and/or voltages to vary, and if this oscillation persists, it will lead to system collapse of the local power grid and connected microgrids.

## 4.3 Automatic Generation Control

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- If the load increases rapidly and the power grid frequency drops, then steam units open the steam valves and hydro unit control loops will open the hydro gates, to supply energy to stabilize the system frequency. This action takes place regardless of the cost of energy from generating units. All units that are under LFC participate in the regulation of the power system frequency. This is called the governor speed control.
- Every 1 to 2 minutes, the supplementary control loop, under AGC, will economically dispatch all units to match load to generation, and at the same time, minimize the total operating cost. Therefore, the AGC will change the set points of the generators under its control. This timing of the cycle can fall within one to several minutes. In Fig. in slide 31, the dotted line section encompasses the AGC, which is located at the local power grid energy - control center.

## 4.4. Operating reserve calculation

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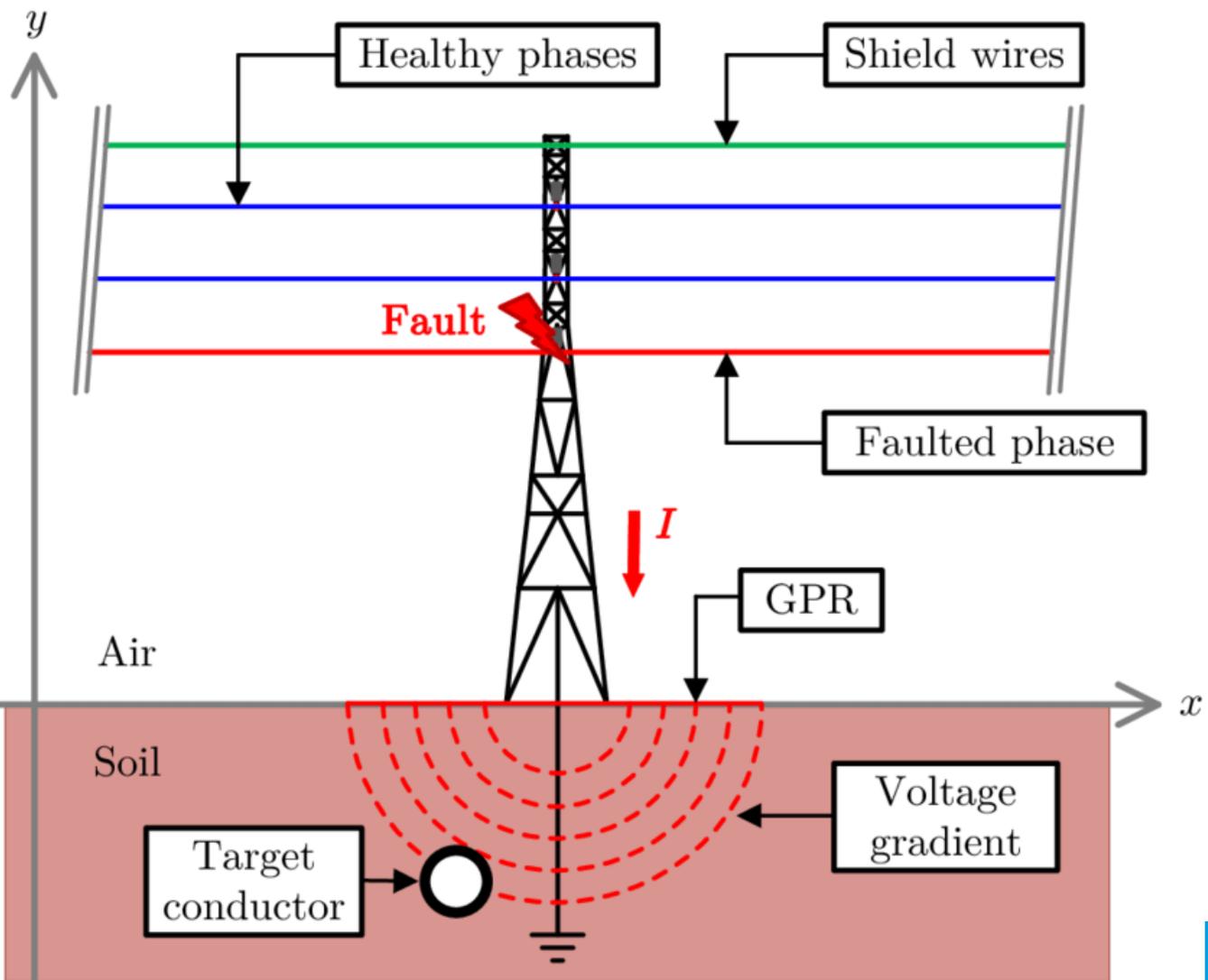
- As discussed, the power grid operation remains stable as long as a balance exists between the system loads and system generation. The operating reserve decision is made based on the security and the necessary reliability. A stable frequency response is essential to stabilize the operation of an interconnected system upon the loss of load or generation outage.
- The spinning reserve is the amount of additional power that is distributed in the form of a few megawatts among many generators operating in the power grid. These units are under AGC control and can dispatch power to ensure the balance of system loads and system generation. The cost of additional power will add to the cost of providing electric energy services.
- The real - time pricing and smart meters will empower many energy end users to participate in providing the spinning reserves in the future operation of power systems, increasing overall efficiency, and reducing the cost of operation of power grids.

## 4.5 Microgrid fault analysis

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- A fault in a power grid is any condition that results in abnormal operation. When energized parts of the system are accidentally connected to the ground, two phase conductors are connected together, or a conductor is broken, the result is a faulted power grid.
- As an example, when a transmission line is accidentally grounded due to weather conditions, such as lightning from an electrical storm, the result is a flashover of the insulation and a flow of high fault currents.
- When a fault or short-circuit occurs in a power grid, all synchronous generators contribute current directly to that fault until protective equipment acts to isolate the fault as quickly as possible. If the fault current is not isolated, the protective system of the power grid will trip (switch-off) the generators, and as a result, the balance between system loads and power generation is lost and the power grid is unstable

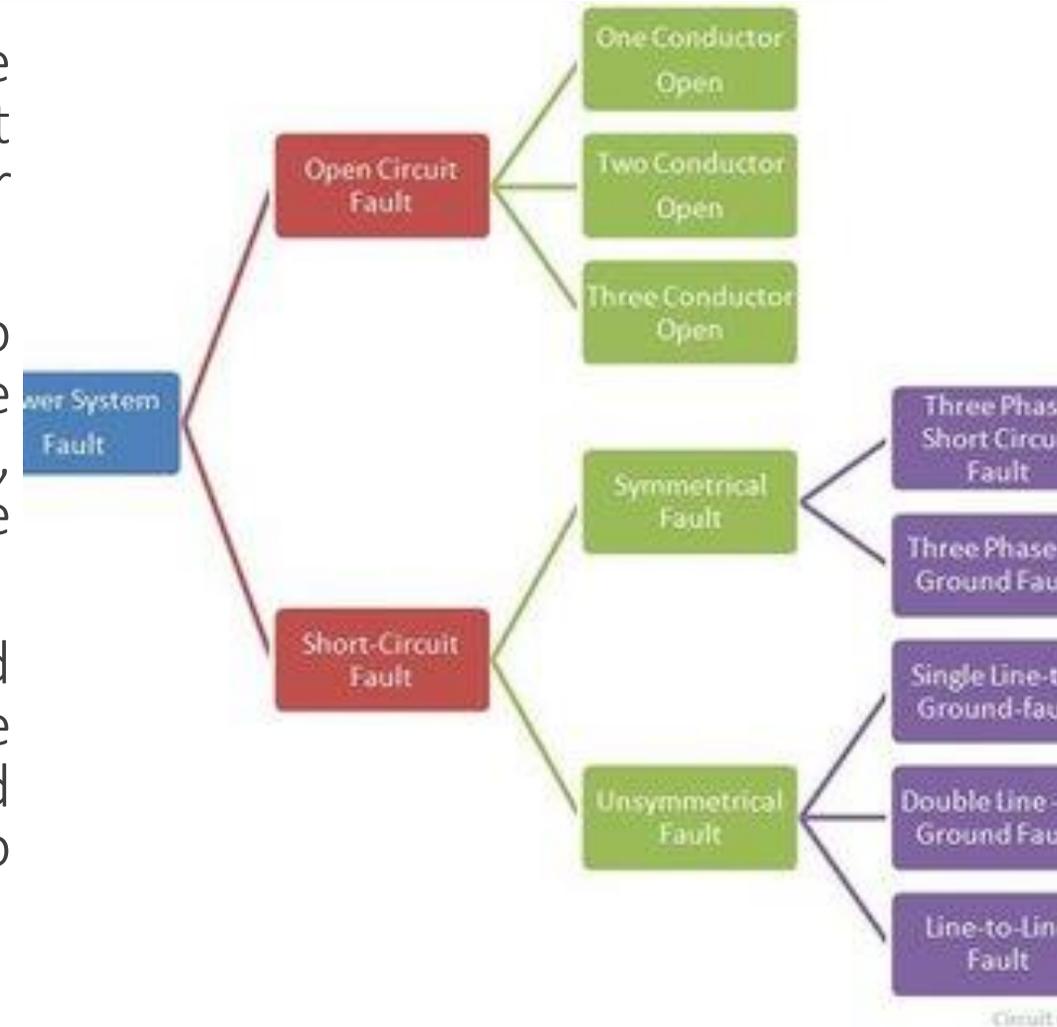
## 4.5 Microgrid fault analysis



Transmission line subjected to a phase-to-ground fault, injecting a current  $I$  into the soil through the tower grounding electrode

## 4.5 Microgrid fault analysis

- The power grid must be designed to operate successfully for the isolation of faults at the highest levels of current that can be anticipated for power grid operation.
- If the fault current exceeds the ability of breakers to extinguish the high fault current and to protect the grid, the result could be a catastrophic failure, fire, and permanent damage to significant portions of the power grid infrastructure.
- Therefore, before microgrids of distributed generation are connected to a local power grid, the fault current contribution must be calculated and mitigating measures must be taken prior to connection.



## 4.5 Microgrid fault analysis

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- In fault studies of power grids, it is assumed that the power grids remain balanced except for the faulted point. Therefore, when a fault occurs, *the power grid must remain balanced*.
- As soon as a fault occurs, the faulted part of the power grid must be quickly isolated and removed from service. Therefore, in fault studies of power grids, we are working with an “if - then condition” : if a point in a power grid is faulted, then we want to calculate the fault current and protect the power grid 's equipment by isolating the faulted part of the system.
- Most faults are a single line to ground or double lines that are faulted and then grounded. For any unbalanced fault current calculation that involves a ground, we use positive, negative, and zero sequence networks. Balanced three - phase faults are also used to size the circuit breakers. For balanced faults, we use the positive sequence network.

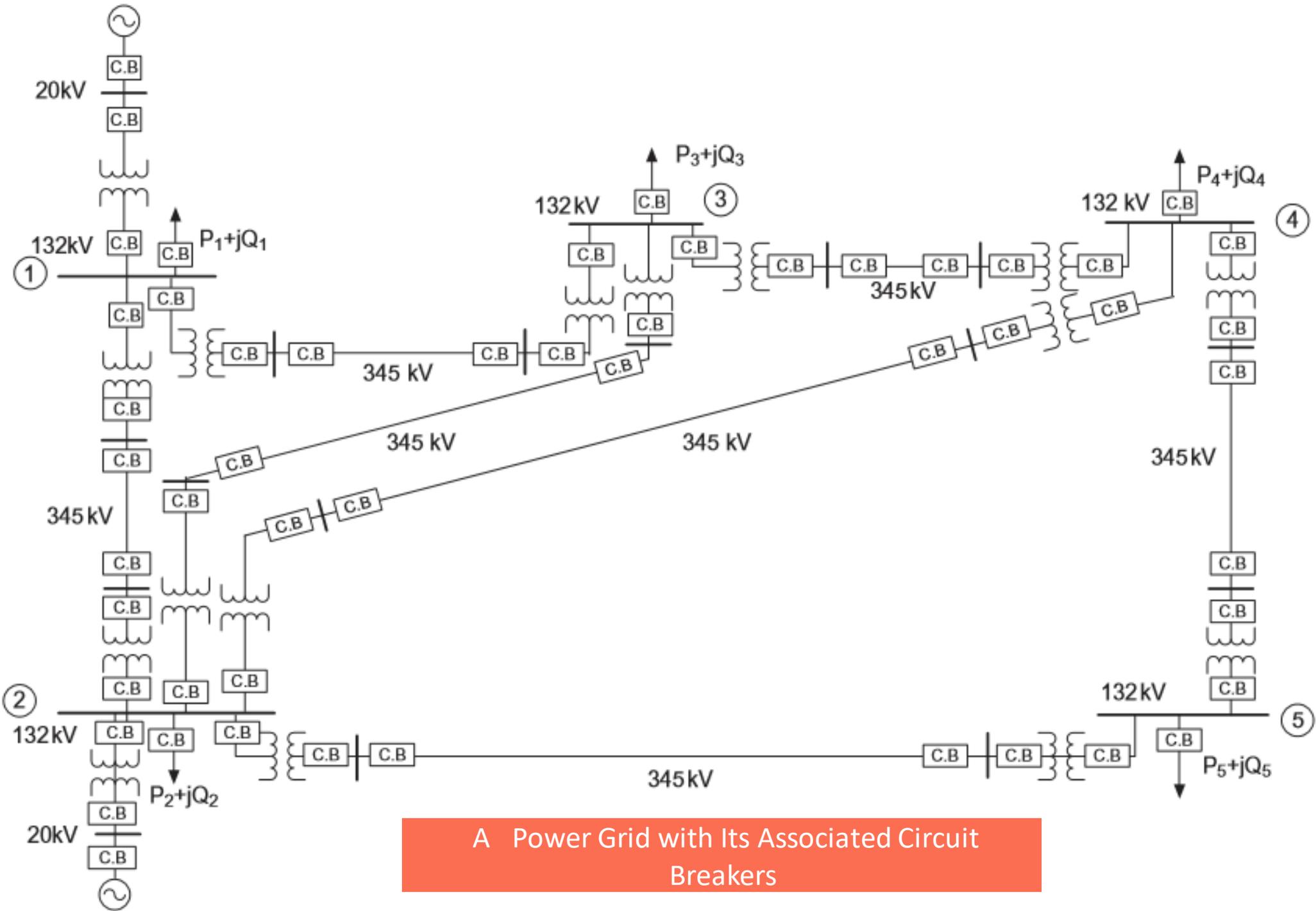
## 4.5.1 Power grid fault current

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- Consider the next Figure below, that depicts a power grid and its circuit breakers.
- The elements of a power grid include generators, transformers, transmission lines, etc. — all of which must be protected so that if a fault occurs the fault currents can be isolated by the circuit breakers.
- For example, if a fault occurs because of a storm, e.g., a line between bus 3 and bus 4 are faulted, both circuit breakers will open by a control action issued from a ground relay fault current detection system.

The main objective of the short-circuit study is to determine the power interruption capability of a circuit breaker at each switching location.

current

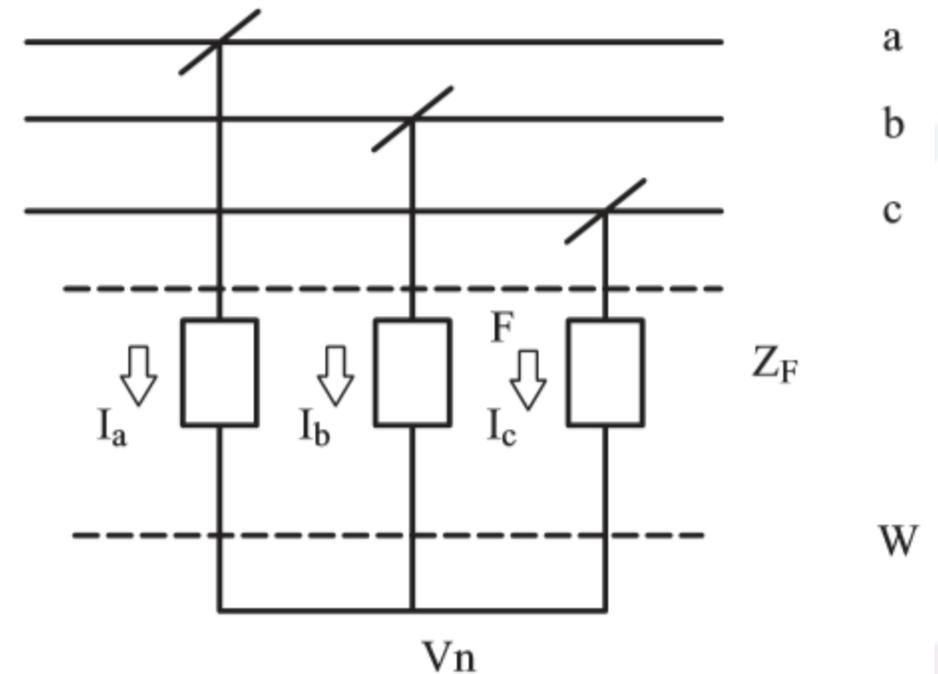


A Power Grid with Its Associated Circuit Breakers

## 4.5.1 Power grid fault current

- To compute the short-circuit current flow through a power grid due to three - phase balanced and unbalanced faults, the power grid system must be modeled to reflect the intended study.
- The types of *faults* are a *three - phase balanced fault* and an *unbalanced fault*.
- Figure depicts a balanced fault at a bus of a power grid where the three phases, a, b, and c are shown. We know that in a *balanced* power grid, the *sum* of phase a, phase b, and phase c *currents adds up to zero*.

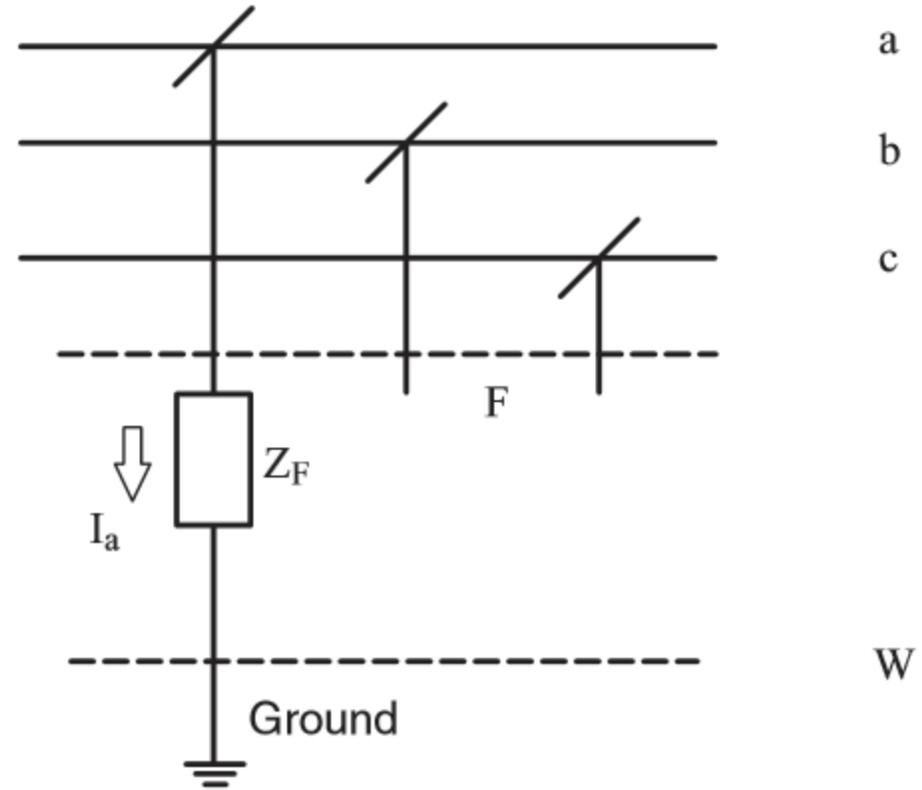
$$I_n = 0$$



A Balanced Three - Phase Fault

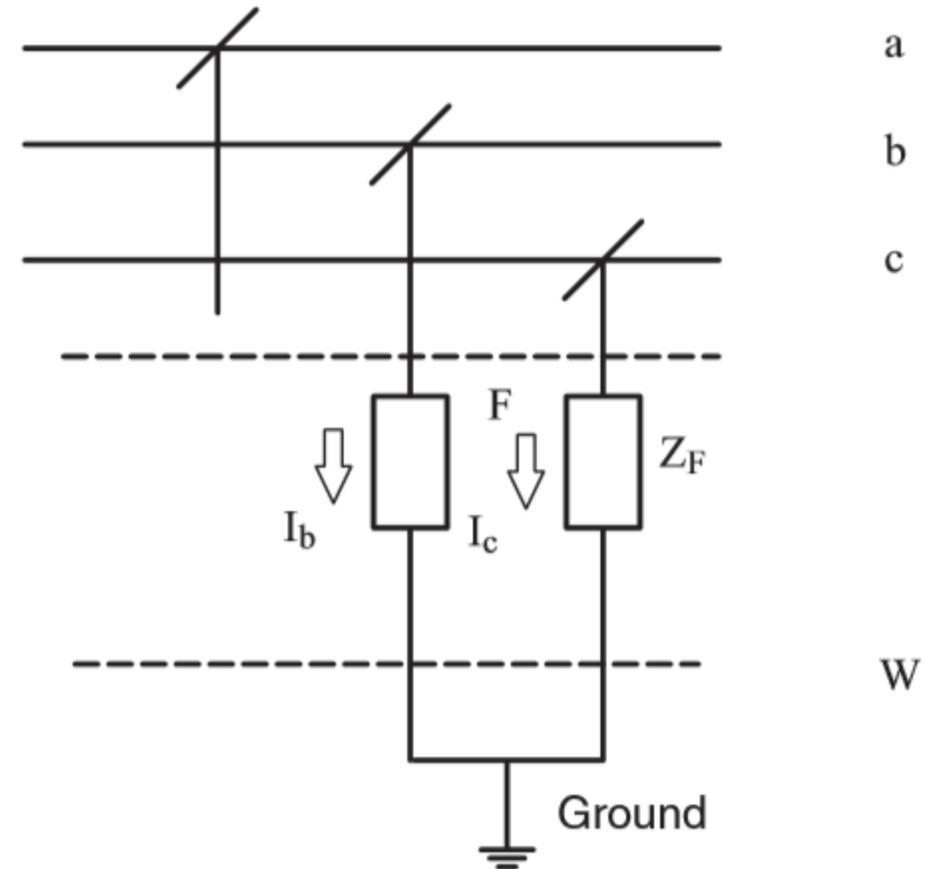
## 4.5.1 Power grid fault current

- Therefore, the *neutral current is zero* and  $I_n = I_a + I_b + I_c$
- If the system is *unbalanced*, the neutral *current will flow through the neutral conductor*. However, if a balanced fault occurs in a balanced power grid, the neutral point where three phases are connected is at zero potential and neutral current will not flow:  $I_n = 0$
- Figure depicts a *single line to ground fault*. For an if - then study of a single line to ground fault, the phase designation is arbitrary. In single line to ground fault studies, it is customary to designate the faulted phase at phase a, with the two other phases operating as normal. Because phase a is faulted, the ground current flow is equal to the fault current of phase a.



## 4.5.1 Power grid fault current

- **Double line to ground fault:** For an if - then study of a double line to ground fault, again, the phase designation is arbitrary. In double line to ground fault studies, it is customary to designate the faulted phases as phases b and c, with phase a operating as normal.
- Because phases b and c are faulted to ground, the ground current flow is equal to the sum of the fault currents of phases b and c.





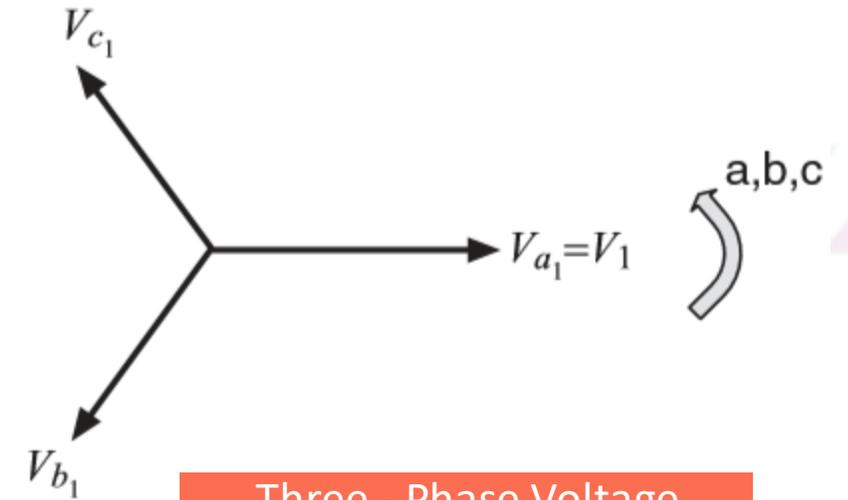
## 4.5.2 Symmetrical components

- The basic concepts of symmetrical components can be introduced by presenting the three - phase systems in terms of positive, negative, and zero sequences [5-7]. Normally, the designation of “1” or “+” is used to present the positive sequence variables of voltage, current, and impedance.
- The balanced three- phase voltages can be expressed as shown in Figure. It also depicts a positive sequence voltage with phase a as the reference followed by phase b and then phase c. The phase voltages, a, b, and c are given by equations

$$V_a = V_{a_1} = V \angle 0^\circ = V_1 \angle 0^\circ$$

$$V_b = V_{b_1} = V_1 \angle 240^\circ$$

$$V_c = V_{c_1} = V_1 \angle 120^\circ$$



Three - Phase Voltage Presented in Terms of Positive Sequence Quantities

## 4.5.2 Symmetrical components

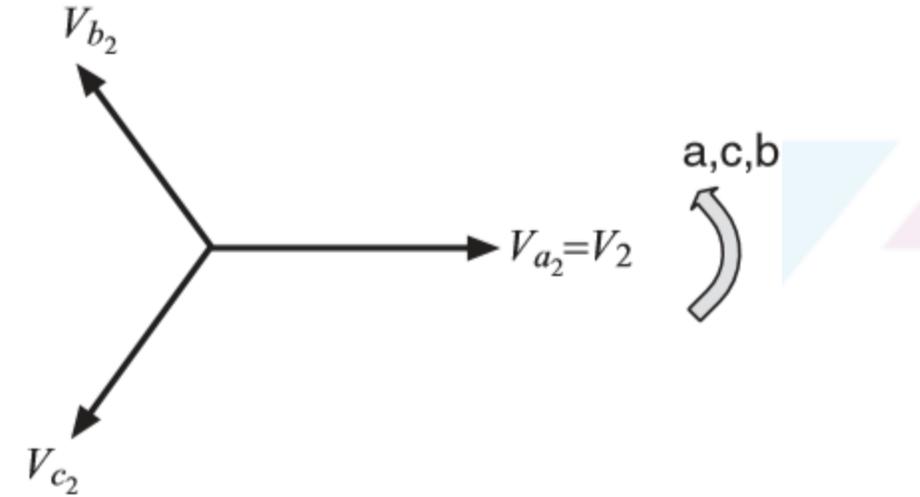
- Let  $\alpha = 1 \angle 120^\circ$  and  $\alpha^2 = 1 \angle 240^\circ$ , then we have

$$V_{a1} = V_1 \quad V_{b1} = a^2 V_1 \quad V_{c1} = a V_1$$

- Rearranging the above in matrix form, we have

$$\begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \end{bmatrix} = \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} V_1$$

- The negative sequence voltage is presented by designating "2", or a negative sign ("-") as a subscript or superscript. The negative sequence is depicted in Figure.



Three-Phase Voltage Presented in terms of Negative Sequence Quantities

## 4.5.2 Symmetrical components

- From above, we may also write in matrix form:

$$\begin{aligned} V_{a_2} &= V_2 \\ V_{b_2} &= aV_2 \\ V_{c_2} &= a^2V_2 \end{aligned} \quad \longrightarrow \quad \begin{bmatrix} V_{a_2} \\ V_{b_2} \\ V_{c_2} \end{bmatrix} = \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix} V_2$$

- The zero sequence voltages are presented by a set of voltages that are in phase. The zero sequence voltages are designated by “0”. The zero sequence voltages are

$$V_{a_0} = V_o$$

$$V_{b_0} = V_o$$

$$V_{c_0} = V_o$$

## 4.5.2 Symmetrical components

- Using the above presentation, a set of three-phase voltages can be expressed in terms of its sequence voltages. In general, a set of unbalanced voltages,  $V_a, V_b, V_c$  can be written as

$$\begin{aligned}
 V_a &= V_{a_0} + V_{a_1} + V_{a_2} \Rightarrow V_a = V_o + V_1 + V_2 \\
 V_b &= V_{b_0} + V_{b_1} + V_{b_2} \Rightarrow V_b = V_o + a^2 V_1 + a V_2 \\
 V_c &= V_{c_0} + V_{c_1} + V_{c_2} \Rightarrow V_c = V_o + a V_1 + a^2 V_2
 \end{aligned}
 \longrightarrow
 \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_o \\ V_1 \\ V_2 \end{bmatrix}$$

- Where the transformation matrix  $T$  can be defined as
- and the voltage can be written as:  $[V_{abc}] = [T][V_{012}]$

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

## 4.5.2 Symmetrical components

- To find the sequence voltages, we multiply by the inverse of  $T$ :

$$[T]^{-1}[V_{abc}] = [T]^{-1}[T][V_{012}]$$

$$[V_{012}] = [T]^{-1}[V_{abc}]$$

$$[T]^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$V_o = \frac{1}{3}(V_a + V_b + V_c)$$

$$V_1 = \frac{1}{3}(V_a + aV_b + a^2V_c)$$

$$V_2 = \frac{1}{3}(V_a + a^2V_b + aV_c)$$

- We can also compute the symmetrical components of current in terms of three - phase currents as:

$$[I_{abc}] = [T][I_{012}]$$

$$[I_{012}] = [T]^{-1}[I_{abc}]$$

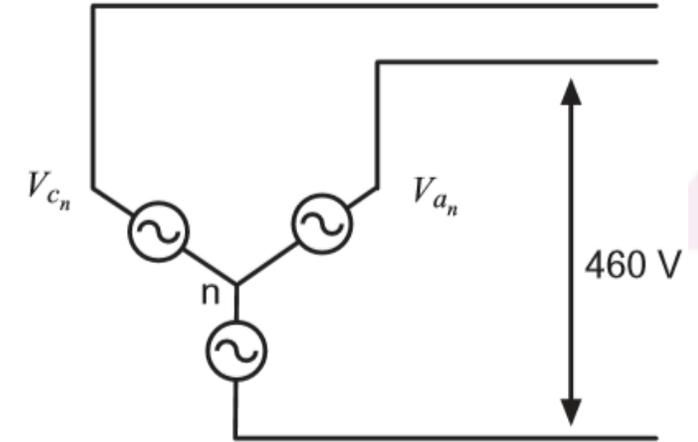
## 4.5.2 Symmetrical components

- **Example 1**
- Consider a balanced, Y - connected, 460V generator. Compute the positive, negative, and zero sequence voltages.
- Let us assume that phase a is selected as the reference phase. The phase a, b, and c voltages are:

$$V_{a_n} = \frac{460}{\sqrt{3}} \angle 0^\circ = 265.9 \angle 0^\circ$$

$$V_{b_n} = 265.9 \angle 240^\circ = 265.9 a^2$$

$$V_{c_n} = 265.9 \angle 120^\circ = 265.9 a$$



$$\begin{bmatrix} V_o \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_{a_n} \\ V_{b_n} \\ V_{c_n} \end{bmatrix}$$

$$\begin{aligned} V_o &= \frac{1}{3} (265.9 + 265.9 a^2 + 265.9 a) \\ &= \frac{265.9}{3} (1 + a + a^2) = 0 \Rightarrow V_o = 0 \end{aligned}$$

## 4.5.2 Symmetrical components

- Since  $(1+a+a^2)=0$   $V_1 = \frac{1}{3}(265.9 + 265.9a^3 + 265.9a^3) = 265.9 \angle 0^\circ$
- Since  $a^4 = a = 1 \angle 120^\circ$  therefore, for the negative sequence voltages we have

$$V_2 = \frac{1}{3}[265.9(1+a+a^2)] = 0$$

- The transformation of the three- phase system to a symmetrical component can be used to show the relationship between the two systems. The three-phase power expressed in terms of phase a, phase b, and phase c can be expressed as

$$S_{3\phi} = [V_a I_a^* + V_b I_b^* + V_c I_c^*]$$

or, in matrix form:  $S_{3\phi} = [V_{abc}]^T [I_{abc}]^*$

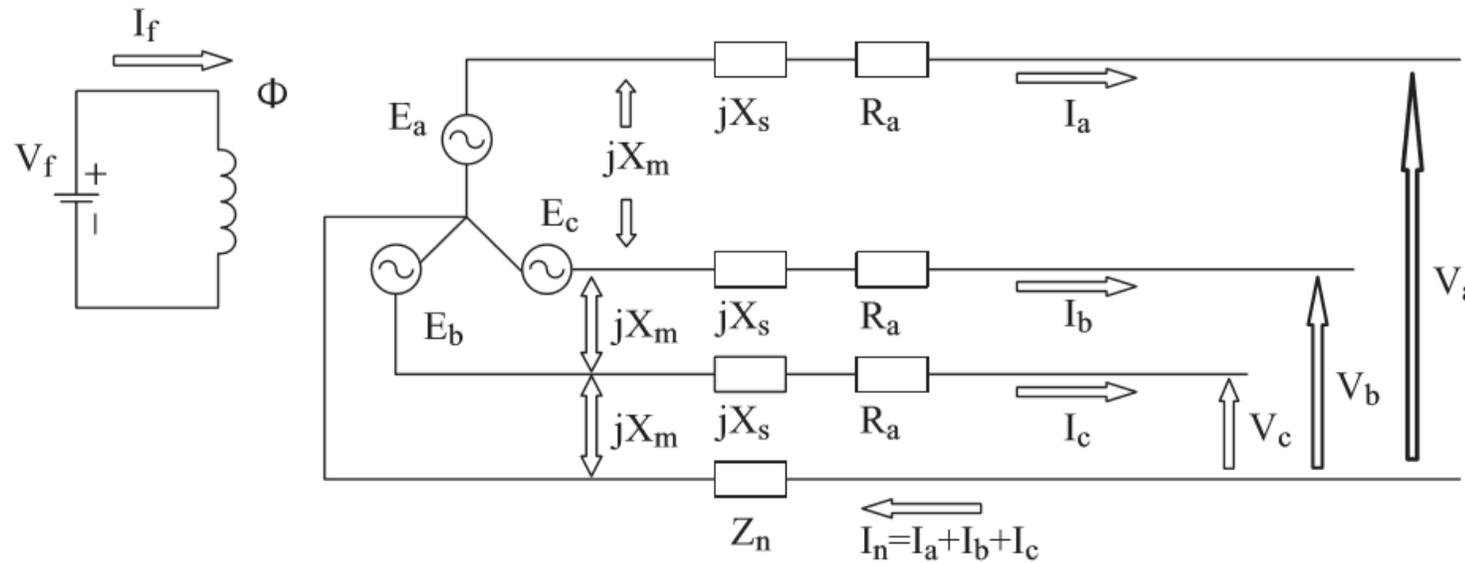
- We can use the symmetrical transformation for both voltage and current and obtain the power in a symmetrical system.

$$S_{3\phi} = [V_{012}]^T [T]^T [T]^* [I_{012}]^*$$

$$S_{3\phi} = 3[V_0 I_0^* + V_1 I_1^* + V_2 I_2^*] = 3[S_{012}]$$

### 4.5.3 Sequence networks for power generators

- Figure depicts an impedance model of a synchronous generator. The impedance,  $Z_n$ , is the grounding impedance. Its function is to limit the ground current fault if a ground fault occurs in the generator.
- The model depicts the steady-state operation of the generator. In this model, the shaft speed,  $\omega_m$  and the field current  $I_f$  are constant. The generator supplies balanced three-phase voltages.



## 4.5.3 Sequence networks for power generators

$$\begin{aligned}
 E_a &= (R_a + jX_s + Z_n)I_a + (jX_m + Z_n)I_b + (jX_m + Z_n)I_c + V_a \\
 E_b &= (R_a + jX_s + Z_n)I_b + (jX_m + Z_n)I_a + (jX_m + Z_n)I_c + V_b \\
 E_c &= (R_a + jX_s + Z_n)I_c + (jX_m + Z_n)I_a + (jX_m + Z_n)I_b + V_c
 \end{aligned} \quad (4.1)$$

- Let us assume that the generator is supplying balanced three - phase voltages. The supply voltage of each phase can be expressed as  $E_a = E$  ,  $E_b = a^2 E$  ,  $E_c = aE$
- We can rewrite the  $Z_s$  and  $Z_m$  as  $Z_s = R_a + jX_s + Z_n$  ,  $Z_m = jX_m + Z_n$
- Then the set of equations given by Equation 4.1 can be written as

$$\begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$[E_{abc}] = [Z_{abc}][I_{abc}] + [V_{abc}] \quad (4.2)$$

## 4.5.3 Sequence networks for power generators

- Substituting  $[I_{abc}] = [T_s][I_{012}]$  in 4.2 and then pre-multiplying by  $[T_s]^{-1}$ :

$$[T_s]^{-1}[E_{abc}] = [T_s]^{-1}[Z_{abc}][T_s][I_{012}] + [T_s]^{-1}[V_{abc}] \quad (4.3)$$

- Recall the transformation from abc to 012 as given below.

$$[E_{012}] = [T_s]^{-1} \cdot [E_{abc}]$$

$$[T_s]^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

- Because the generator is supplying balanced three- phase voltages, the right- hand side of Equation 4.3, can be written as the generator sequence voltages as given by Equation 4.4:

$$\begin{bmatrix} E_0 \\ E_1 \\ E_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} E \\ a^2 E \\ aE \end{bmatrix} = \frac{E}{3} \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ E \\ 0 \end{bmatrix}$$

## 4.5.3 Sequence networks for power generators

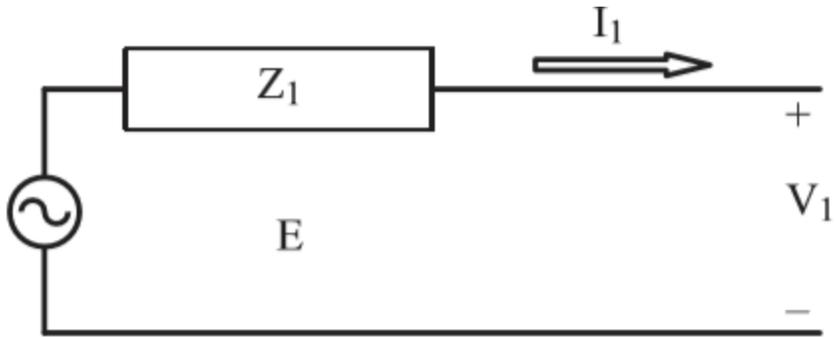
- We can rewrite the expression  $[T]^{-1}[Z_{abc}][T] = [Z_{012}]$  as:

$$[T]^{-1}[Z_{abc}][T] = [Z_{012}] = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$$

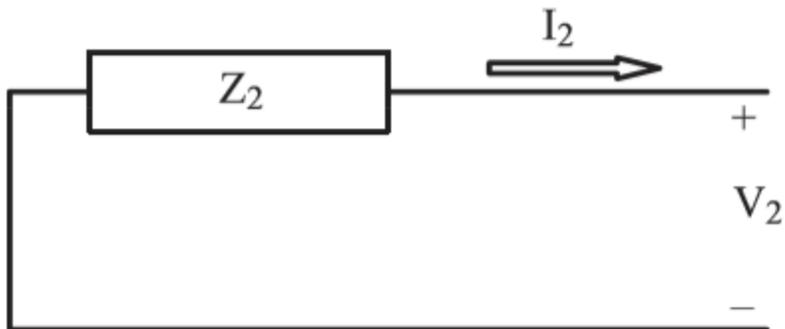
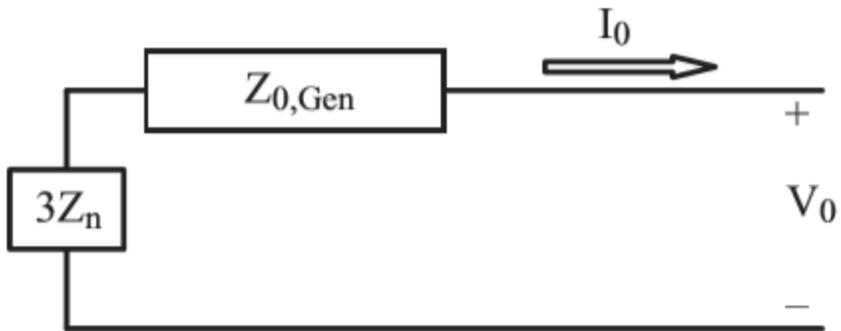
- Where  $Z_0 = Z_{0,Gen} + 3Z_n$  and  $Z_{0,Gen} = R_a + j(X_s + 2X_m)$   $Z_0 = R_a + j(X_s + 2X_m) + 3Z_n$   
 $Z_1 = Z_s - Z_m = R_a + j(X_s - X_m)$   $Z_2 = Z_s - Z_m = R_a + j(X_s - X_m)$
- Eq. (4.3) can be written as

$$\begin{bmatrix} 0 \\ E \\ 0 \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

## 4.5.3 Sequence networks for power generators



Positive, Zero, and  
Negative Sequences of  
a Generator



## 4.5.3 Sequence networks for power generators

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- The previous system of equations will result in the following sequence of zero, positive, and negative networks

$$Z_0 I_0 + V_0 = 0$$

$$Z_1 I_1 + V_1 = E$$

$$Z_2 I_2 + V_2 = 0$$

- Therefore, when the three - phase generator supplies balanced three - phase voltages, only the positive sequence network is excited by positive sequence voltage, that is the same as phase a of the three - phase system.

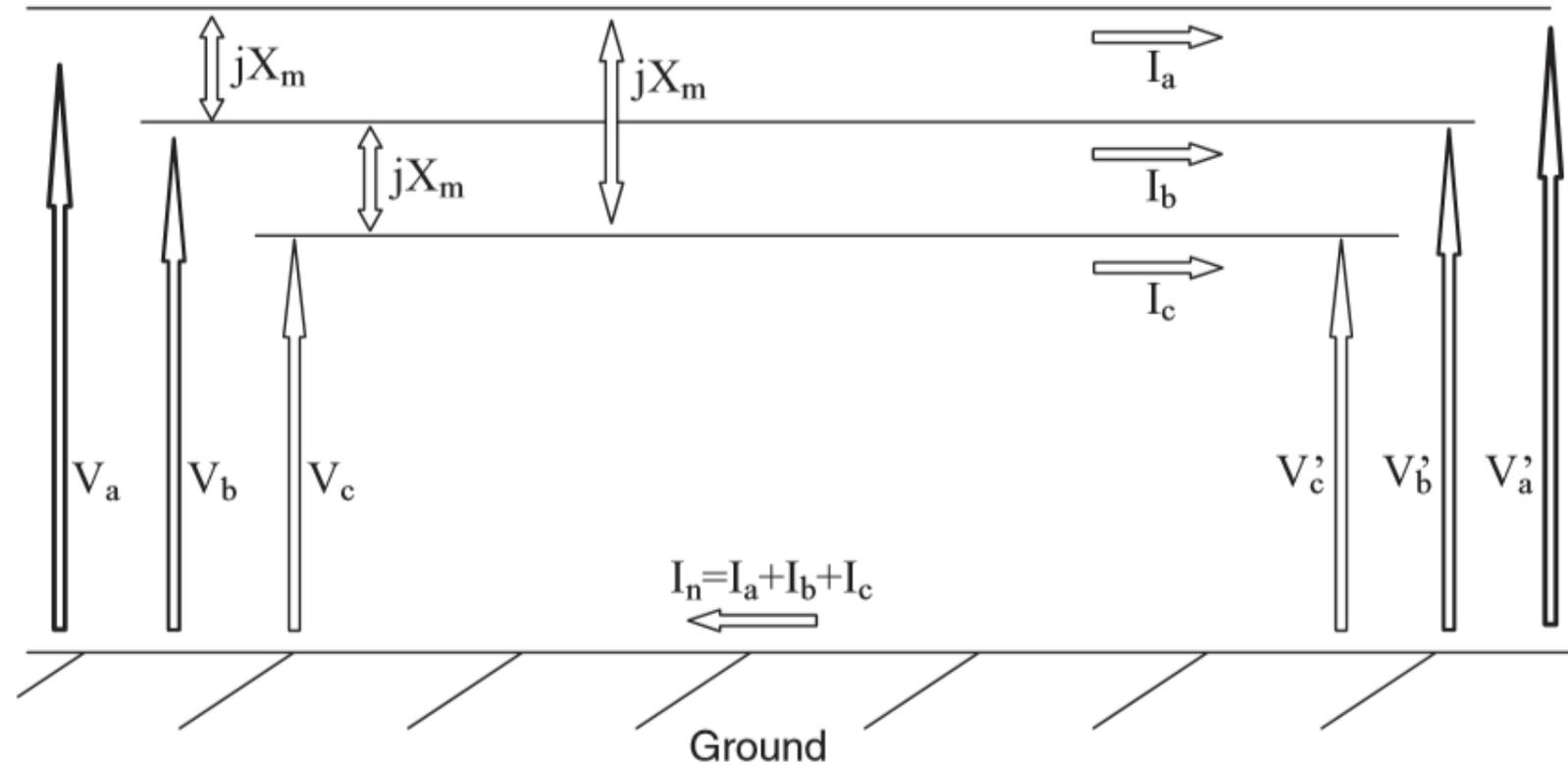
## 4.5.4 Sequence networks for balanced three-phase lines

The Figure depicts the balanced three-phase network model of a transmission line. The voltage equations expressing the voltage drop across the lines can be expressed as given by 4.4:

$$V_a = jX_s I_a + jX_m I_b + jX_m I_c + V'_a$$

$$V_b = jX_s I_b + jX_m I_a + jX_m I_c + V'_b$$

$$V_c = jX_s I_c + jX_m I_a + jX_m I_b + V'_c$$



## 4.5.4 Sequence networks for balanced three-phase lines

- In matrix form:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \begin{bmatrix} V'_a \\ V'_b \\ V'_c \end{bmatrix} = j \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$[V_{abc}] - [V'_{abc}] = [Z_{abc}][I_{abc}], \quad [V_{abc}] = [T_s][V_{012}], \quad (4.5) \quad [I_{abc}] = [T_s][I_{012}] \quad (4.6)$$

- Replace  $V_{abc}$  with Equation 4.5 and  $I_{abc}$  with Equation 4.6 to obtain:

$$[T_s][V_{012}] - [T_s][V'_{012}] = [Z_{abc}][T_s][I_{012}] \quad (4.7)$$

- Multiplying Equation 4.7 by  $[T_s]^{-1}$

$$[T_s]^{-1}[T_s][V_{012}] - [T_s]^{-1}[T_s][V'_{012}] = [T_s]^{-1}[Z_{abc}][T_s][I_{012}] \quad (4.8)$$

## 4.5.4 Sequence networks for balanced three-phase lines

- Since  $[T_s]^{-1}[T_s] = 1$  we get  $[V_{012}] - [V'_{012}] = [Z_{abc}][I_{012}]$

$$[Z_{012}] = [T_s]^{-1}[Z_{abc}][T_s]$$

- $$Z_{012} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} j \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \quad (4.10)$$

- Eq. (4.10) simplifies as

$$Z_{012} = j \begin{bmatrix} X_s + 2X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s - X_m \end{bmatrix}$$

## 4.5.4 Sequence networks for balanced three-phase lines

- Therefore, the symmetrical sequence network model of a transmission line can be expressed as

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} - \begin{bmatrix} V_0' \\ V_1' \\ V_2' \end{bmatrix} = j \begin{bmatrix} X_s + 2X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s - X_m \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

- And zero, positive, and negative sequence model networks are

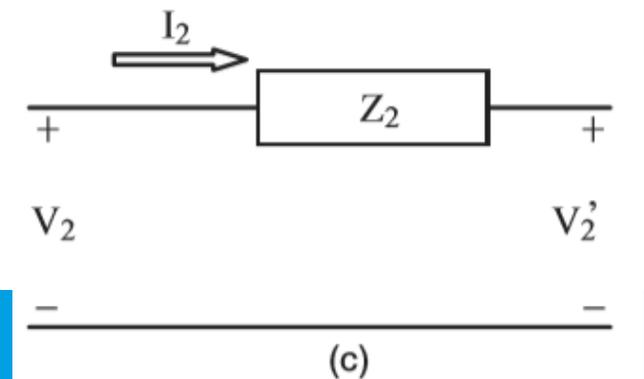
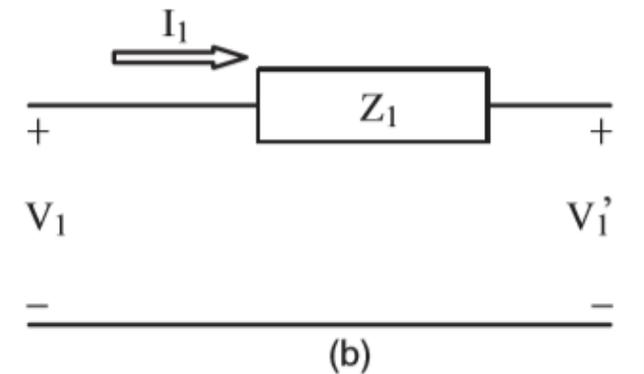
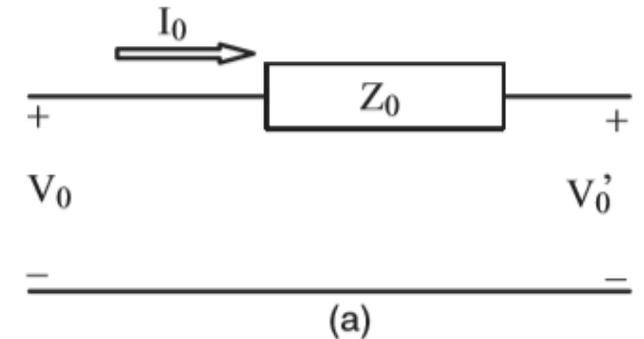
$$Z_0 = \text{Zero Sequence Impedance} = j(X_s + 2X_m)$$

$$Z_1 = \text{Positive Sequence Impedance} = j(X_s - X_m)$$

$$Z_2 = \text{Negative Sequence Impedance} = j(X_s - X_m)$$

## 4.5.4 Sequence networks for balanced three-phase lines

- Figure depicts the sequence networks for a transmission line's zero, positive, and negative circuit models.



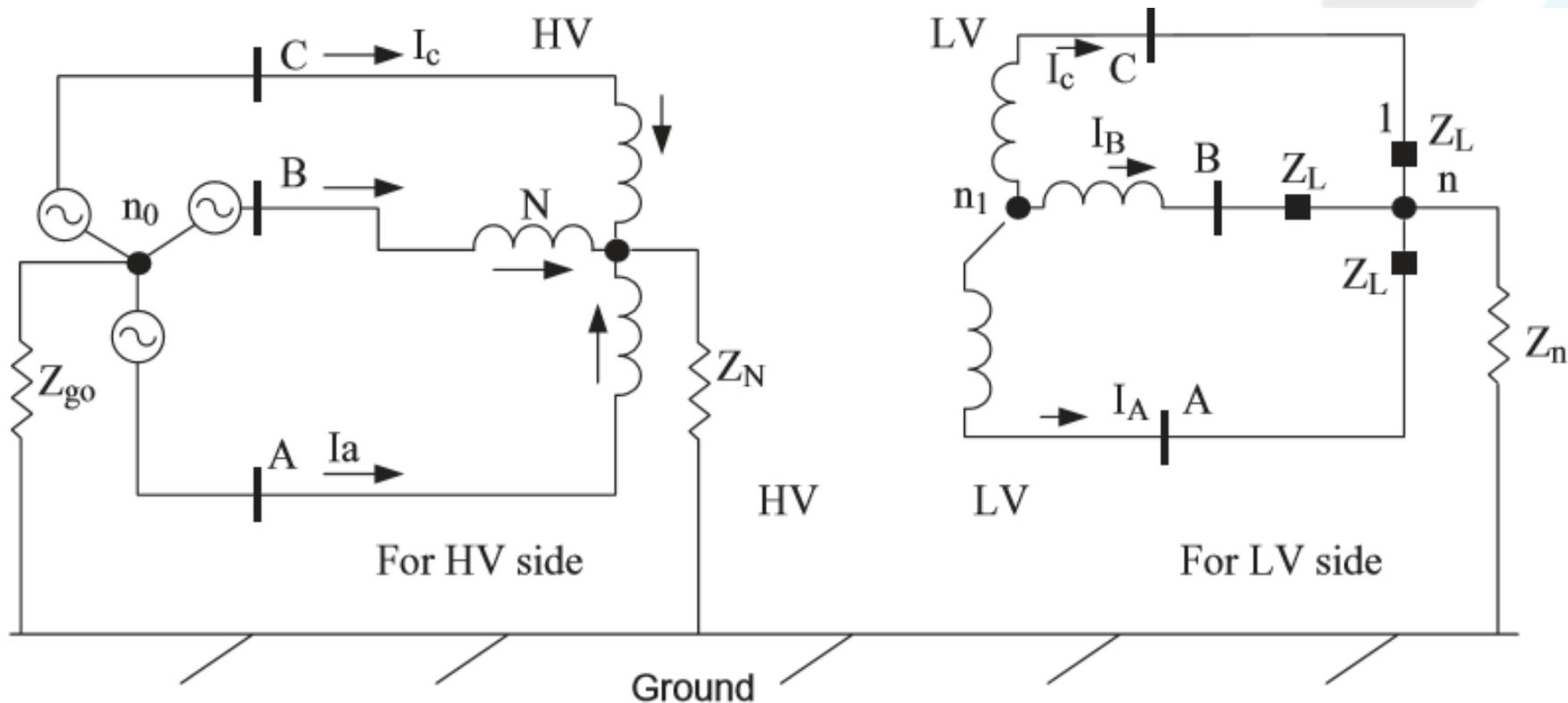
## 4.5.5 Ground current flow in balanced three-phase transformers

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- The neutral point of a power grid is often connected to earth ground. Often ground and neutral are the same electrical point if there is no ground impedance between neutral and the earth ground point.
- The conductor that connects the power grid's neutral point and ground will not carry load current; its function is to detect the ground fault current. If a power grid is faulted and the faulted bus is connected to ground, then the first question of interest is how ground current fault will flow through the power grid.
- The transformers' high and low voltage sides are not electrically connected. The voltages induced in either side of the transformers are due to magnetic coupling of respective windings.

## 4.5.5 Ground current flow in balanced three-phase transformers

- Consider a Y-Y - connected transformer in which the three - phase voltages supplied to the three-phase transformers are not balanced:



## 4.5.5 Ground current flow in balanced three-phase transformers

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- The question is whether ground current can flow in the transformer when it is grounded on the generator side (high voltage side) and its low voltage side is not grounded.
- To answer this question, we need to remember that in transformers, voltage is induced in the low voltage side by magnetic induction and resulting currents that are flowing must obey the Kirchhoff current law.
- This means the current must return to its generating source. Therefore, the ground current cannot flow in either the high voltage side or the low voltage side. If the ground current flows in the low voltage side, it must return to neutral on the low voltage side. However, if the low voltage side is not grounded, it cannot complete the flow path.

## 4.5.5 Ground current flow in balanced three-phase transformers

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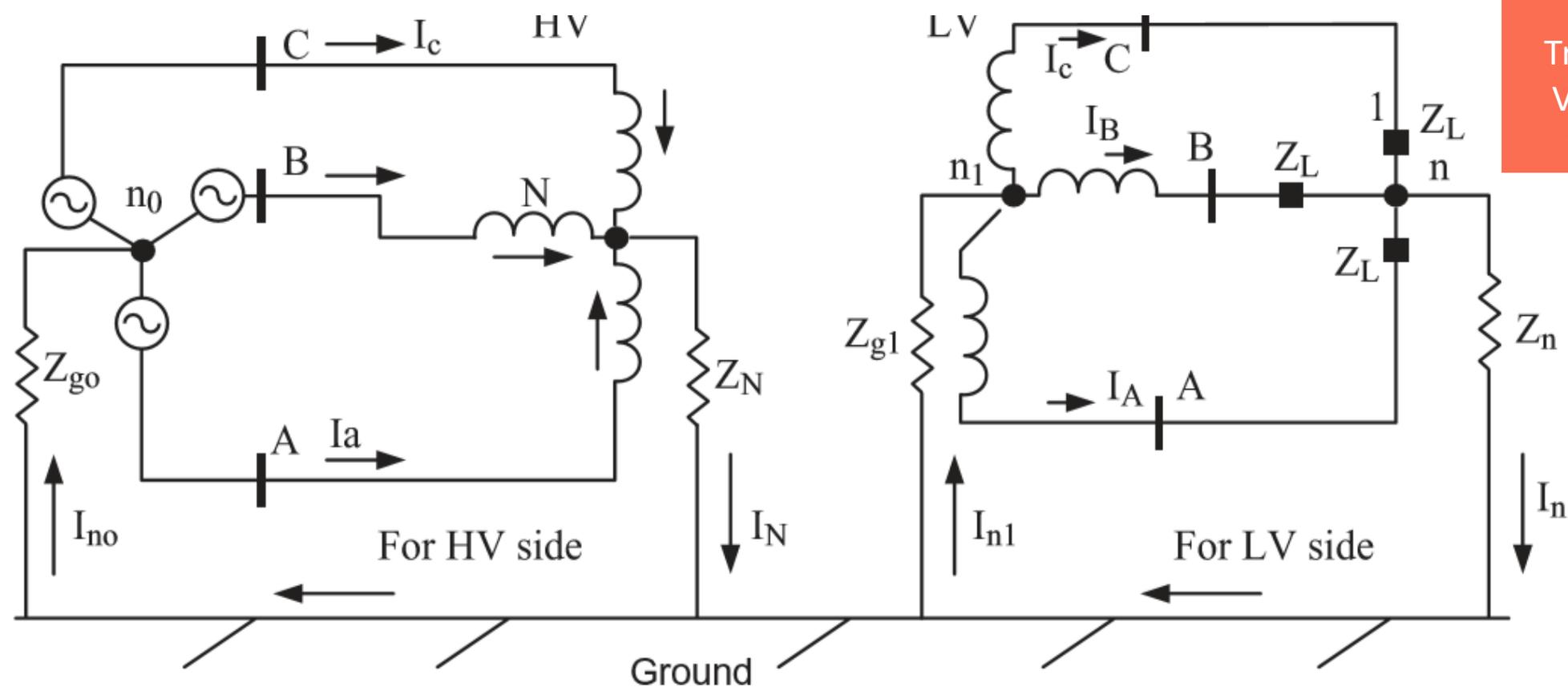
- Therefore, if there is no flow path for the ground current, then the low voltage side neutral will be at a value that will satisfy the following:

$$V_n = V_{an} + V_{bn} + V_{cn}$$

- By the same reasoning, the ground current cannot flow on the high voltage side. If there is ground current flowing on the high voltage side, the magnetic coupling must induce the three - phase voltages on the low side and have the low - side phase current flowing, which will add up and result in ground current flow on the low voltage side.
- However, the low voltage side is not grounded; therefore, the low- side voltages will add up as given in Equation above.



# 4.5.5 Ground current flow in balanced three-phase transformers

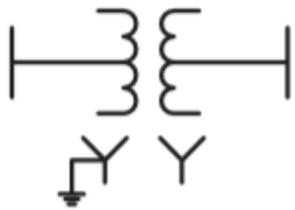


The Y-Y - Connected Transformer with the High Voltage Side and the Low Voltage Side Grounded

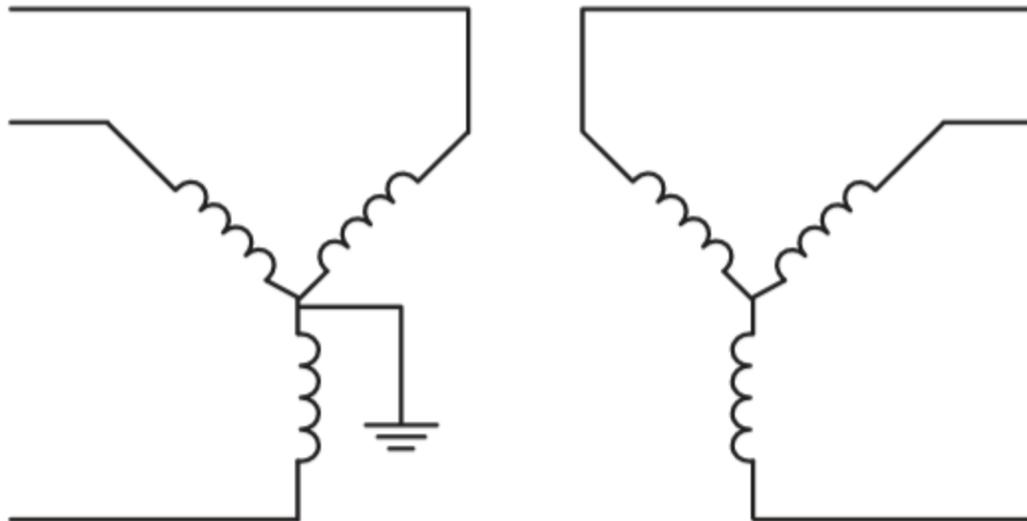
## 4.5.6 Zero sequence networks

- For a Y - Y transformer with grounded neutral on one side and ungrounded neutral on the other side, the ground current cannot flow because there is no ground path on the ungrounded side as shown below.

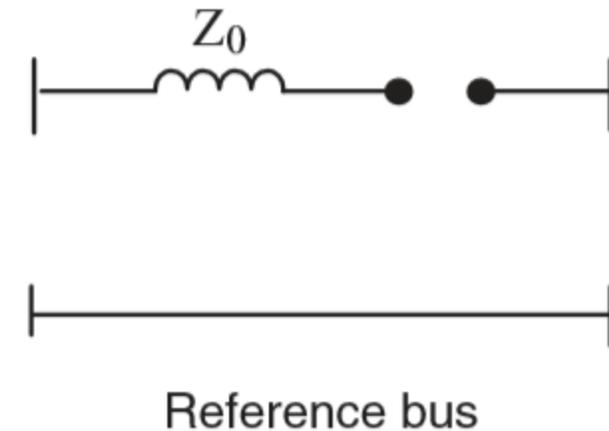
Symbols



Connection Diagrams



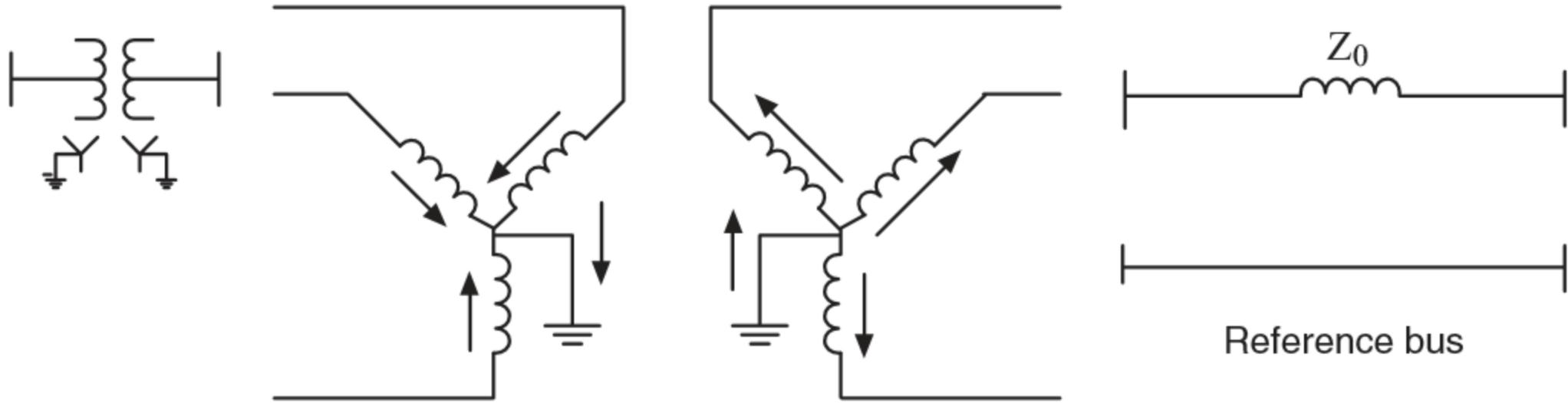
Zero-Sequence Equivalent Circuits



## 4.5.6 Zero sequence networks

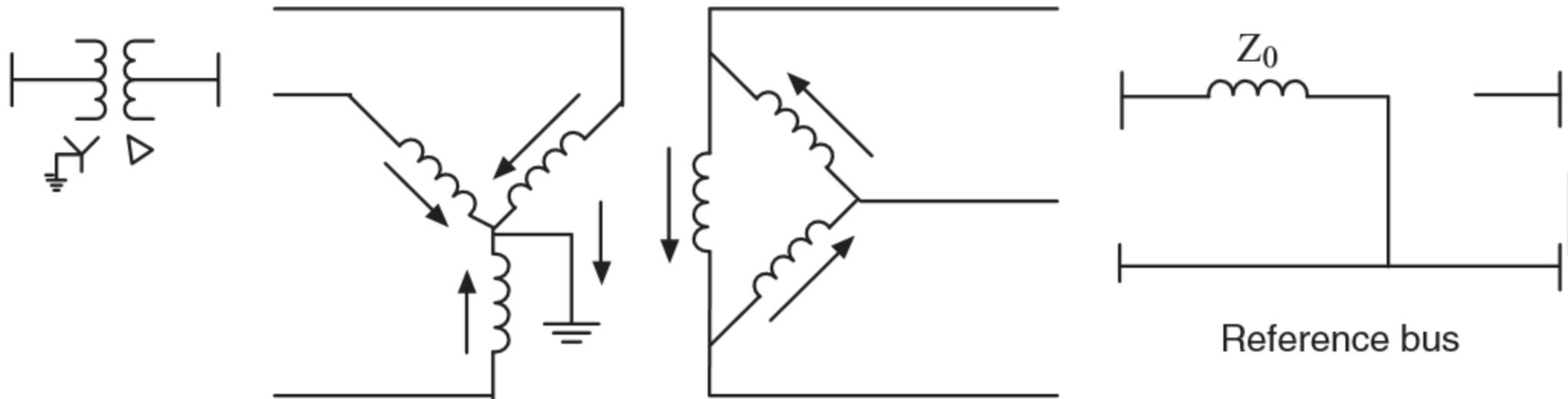
- Ground current can flow if the neutral point of both sides of a Y-Y - connected transformer is grounded as shown in Fig. in the next slide. The arrows show the direction of the current flow and resulting ground current,  $I_n$ .
- Figure displays a transformer that has been supplied with unbalanced voltages on one side that results in the current  $I_n$  because for the unbalanced three-phase currents  $I_n = I_a + I_b + I_c$ . As shown in Fig., the ground current  $I_n$  flows through the ground conductor; the arrow is pointing downward. On the other side of the transformer the unbalanced voltages result in a ground current flowing out of the ground so that  $I_n = I_a + I_b + I_c$ .
- Of course, the relationship of ground currents on both sides of the transformer is governed by the turn ratio of the transformer.
-

## 4.5.6 Zero sequence networks



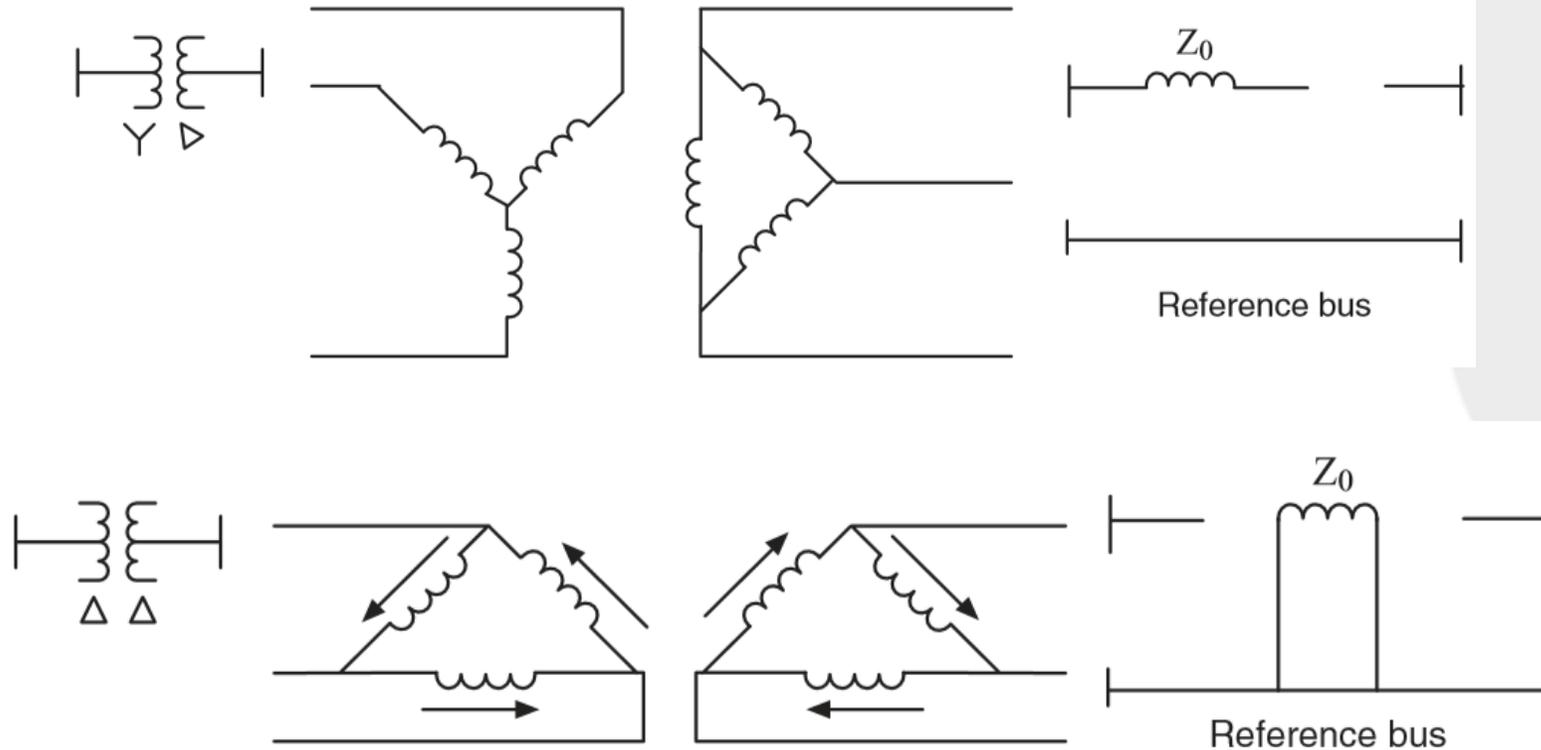
## 4.5.6 Zero sequence networks

- Next Figure depicts a grounded Y- $\Delta$  transformer. In this case, the ground current flows on the grounded Y side because we have a circulating current on the  $\Delta$  side of the transformer.
- Again, we should remember that if unbalanced three - phase voltages are applied to the grounded Y side, we have created a ground path for ground current to flow.



## 4.5.6 Zero sequence networks

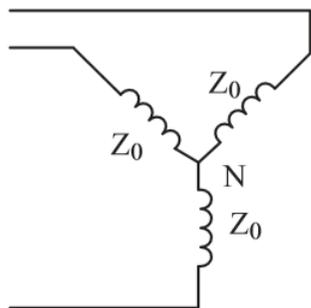
- Figures below depict the conditions of an ungrounded Y- $\Delta$  connection and  $\Delta$ - $\Delta$  connection.



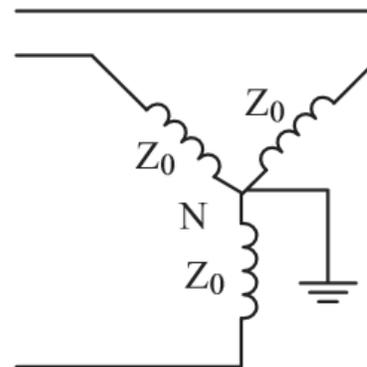
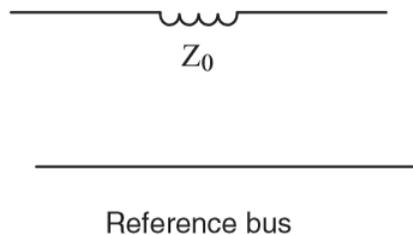
## 4.5.6 Zero sequence networks

- Figure (a) (next slide) shows the zero sequence of a Y- connected load when the load is not grounded. As seen from the zero sequence of the ungrounded load, no ground current flows.
- In Fig. (c) (next slide) , three times the value of the ground impedance  $Z_n$  appears in the zero sequence network for the grounded load because  $I_n = I_o$ .
- Finally, Fig. (d) (next slide), depicts the  $\Delta$ -connected load. In this case, the zero sequence current circulates as shown in Fig (d).

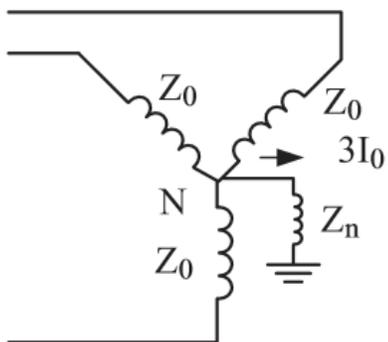
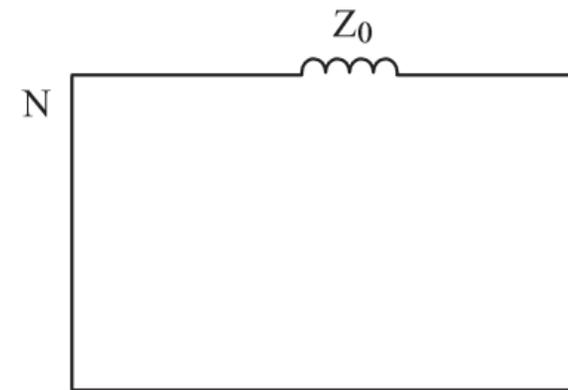
# 4.5.6 Zero sequence networks



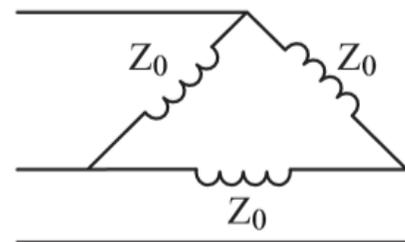
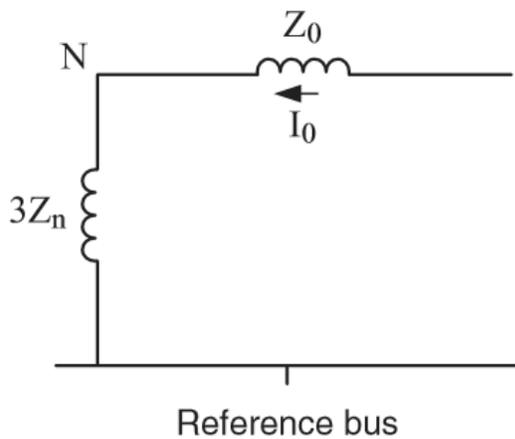
(a)



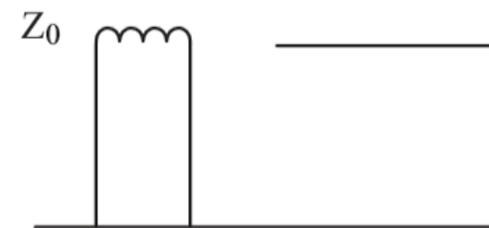
(b)



(c)



(d)



## 4.5.7 Fault calculations

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- For a *balanced three* - phase fault, only the *positive sequence* network is *excited*. For *unbalanced* faults, *all three sequence* networks *may be excited*.
- If a fault involves a connection to ground, the ground current will flow. The back emf voltage behind the generator reactance is assumed equal in magnitude and phase angle; normally, it is assumed to be equal to 1 per unit.
- For a faults study, normally, all shunt elements including loads and line charging are neglected. Loads may be represented with constant load impedance models. All tap changing transformers are assumed to be at their nominal tap settings and balanced transmission lines are assumed. The negative and positive sequence networks are assumed equal; **only** coupling between adjacent circuits is taken into account in a zero sequence network.

## 4.5.7 Fault calculations

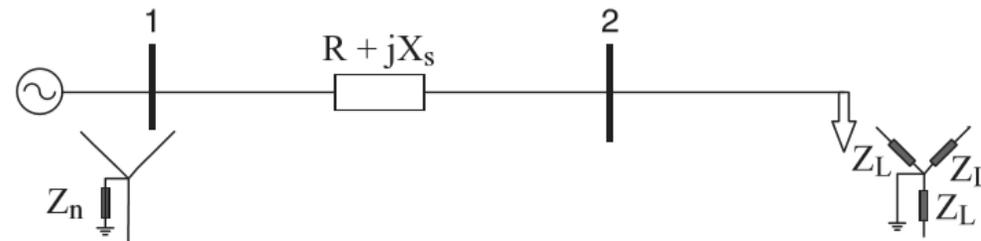
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- The study of unbalanced faults requires the modeling of the power grid using the symmetrical analysis of a power grid. Therefore, we use the sequence network of generators, transformers, transmission lines, and loads. Based on the type of unbalanced faults, one line to ground, two lines to ground, line to line faults, etc., the symmetrical models of the power grid are constructed. These sequence network models are used to construct the unbalanced fault models of the power grid.
- The objectives of balanced fault studies are to determine the required circuit breaker short capacity in kVA or MVA. The objectives of unbalanced fault studies are to determine how to set the protective relay systems.

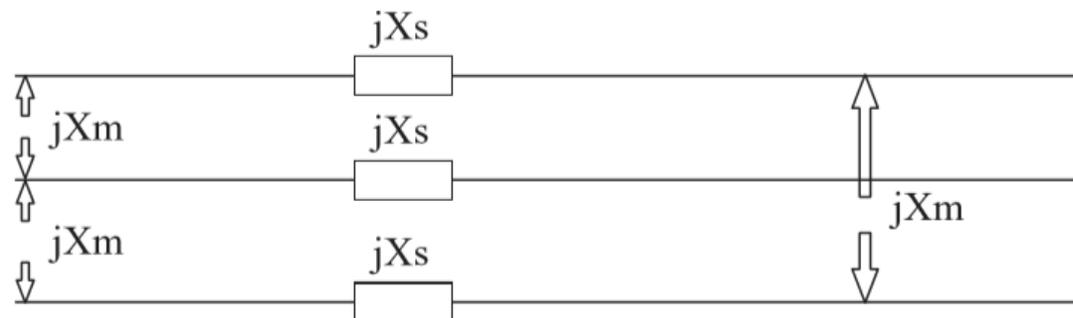
## 4.5.7 Fault calculations

### Example 2

- Consider the power grid given below. Assume the generator positive, negative and zero sequence impedances are  $Z_{gen,1}$ ,  $Z_{gen,2}$ , and  $Z_{gen,0}$ , the generator is grounded through the ground impedance  $Z_n$ .



- Also, assume the transmission line model as depicted below:



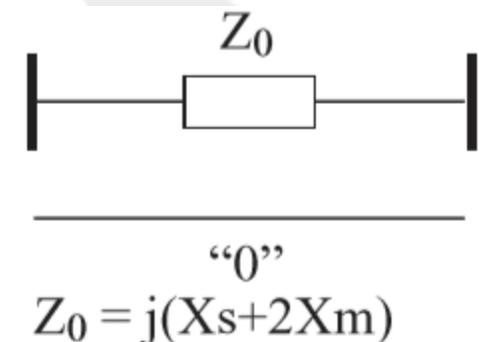
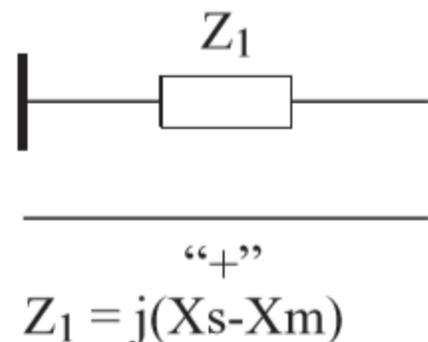
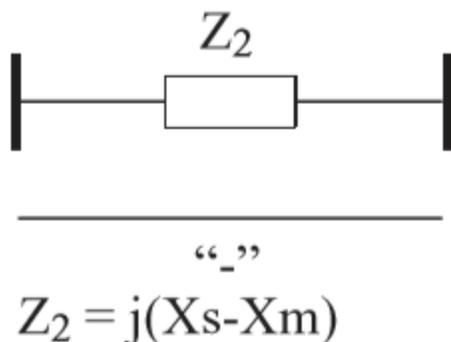
## 4.5.7 Fault calculations

### Example 2

i) If the supply generator is unbalanced and supplies three - phase unbalanced voltages, determine the positive, negative, and zero sequence networks for the one - line diagram

ii) If the supply generator is balanced and supplies three - phase balanced voltages, determine the positive, negative, and zero sequence networks for the one - line diagram

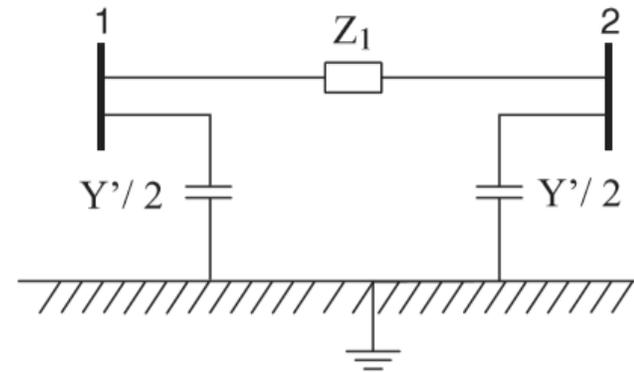
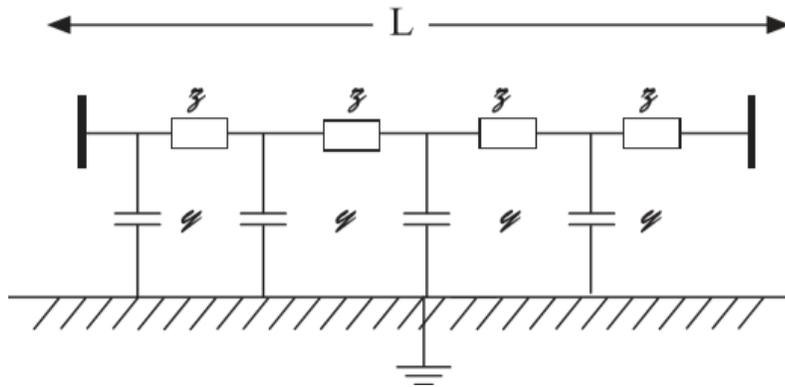
- The positive, negative, and zero sequence of the transmission line is presented as follows:



## 4.5.7 Fault calculations

### Example 2

- If the transmission line has the total length of “L”, the distributed line impedance and line charging capacitance and its lumped model is shown below

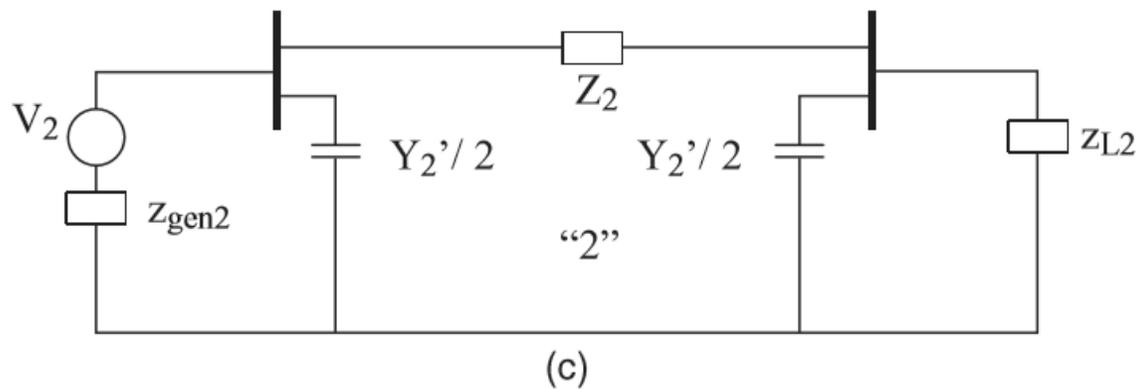
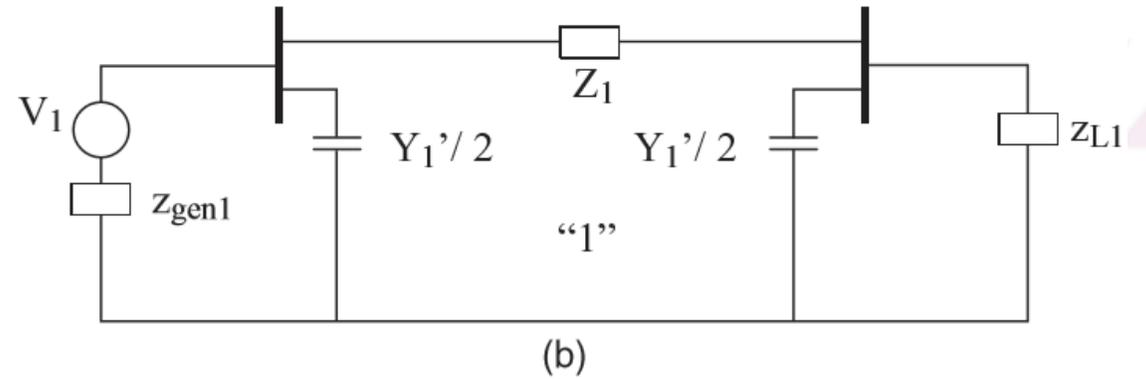
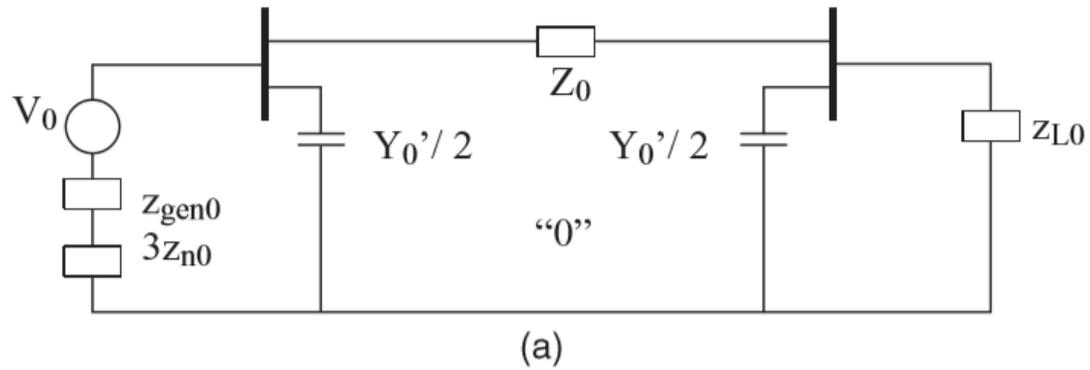


$$z_1 = L \times z$$

$$Y' = L \times y$$

## 4.5.7 Fault calculations

### Example 2

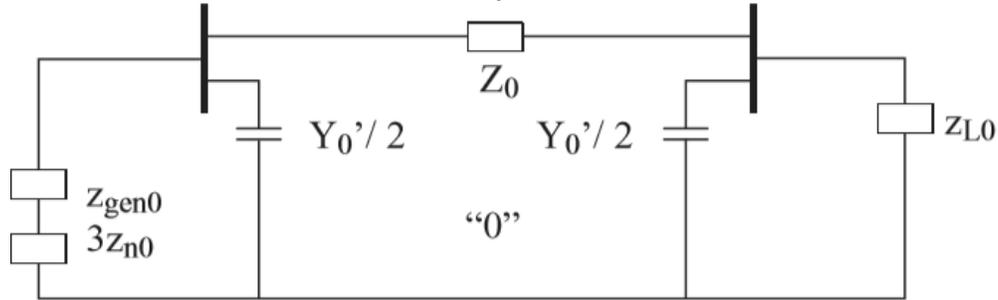


(a) The Zero Sequence, (b) Positive Sequence, and (c) Negative Sequence when the Power Supply Generator Voltages Are Unbalanced.

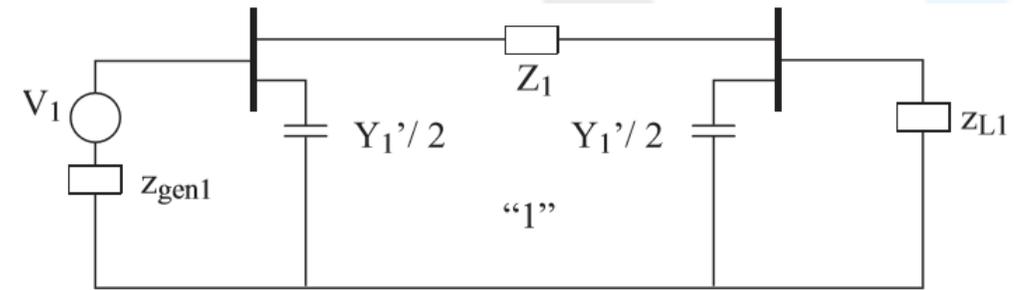
## 4.5.7 Fault calculations

### Example 2

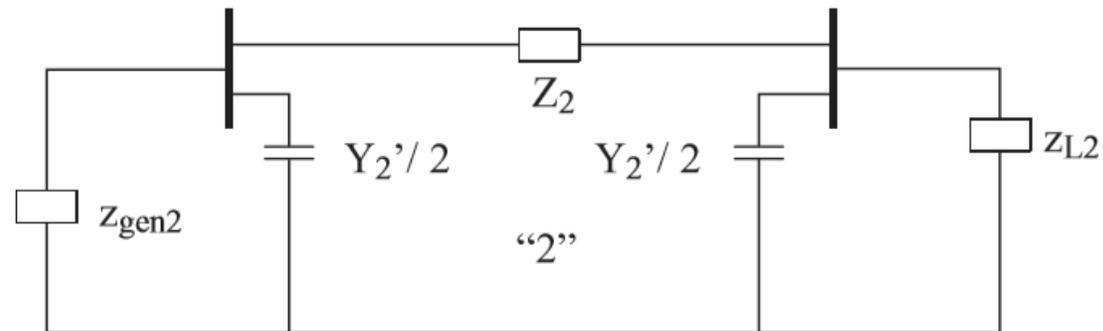
If the supply generator is balanced, the zero, positive, and negative sequence networks are as depicted in the next Fig.



(a)



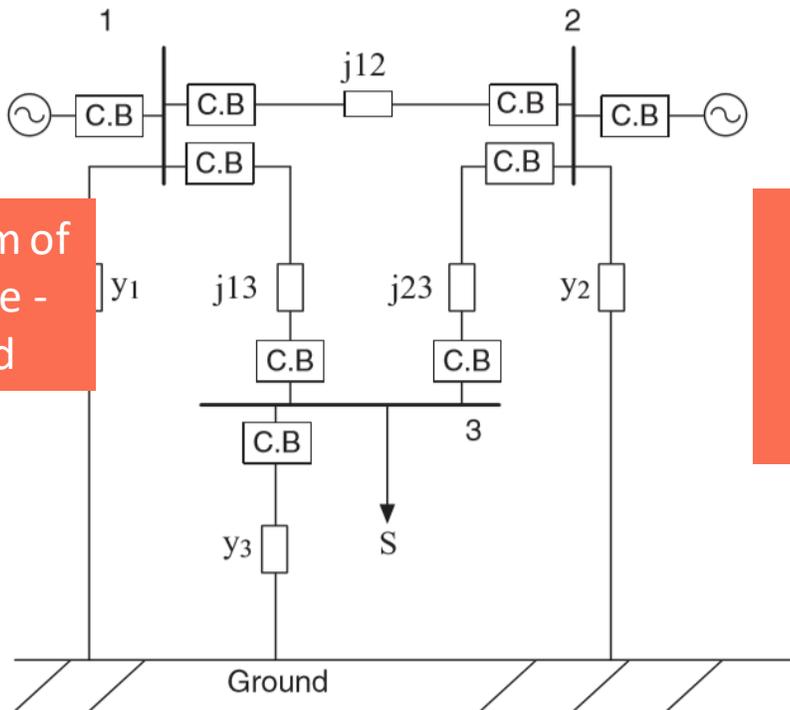
(b)



(c)

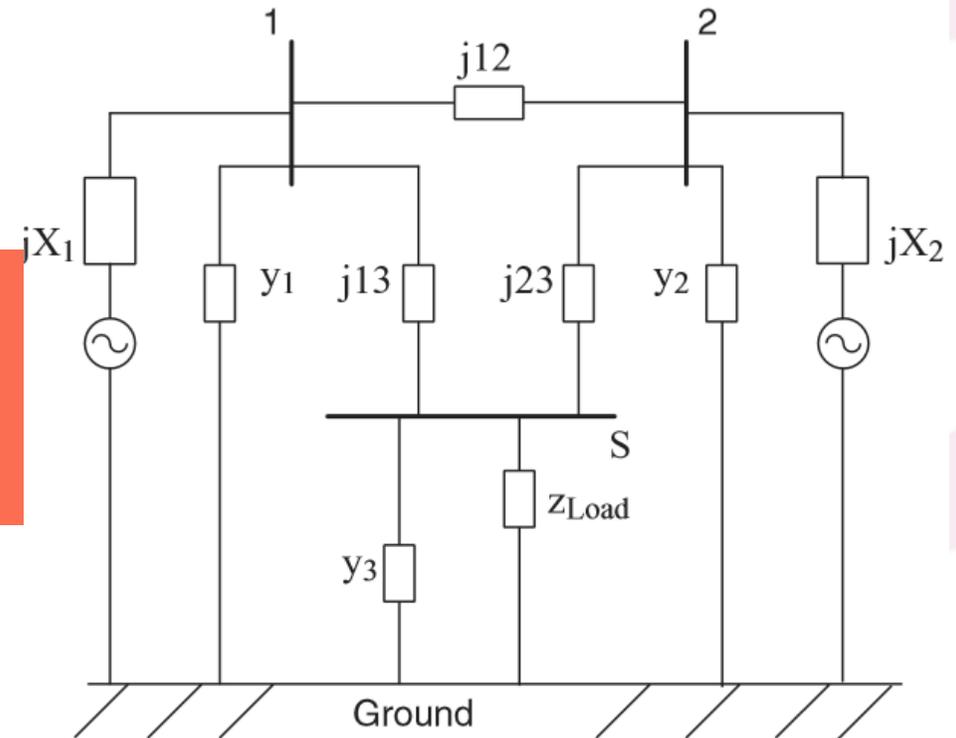
## 4.5.7 Balanced three-phase faults calculations

- For a balanced three-phase faults study, only the positive network model must be constructed. Figures below depict a one-line diagram of a three-bus power grid and the positive sequence network model for balanced fault studies including the shunt elements and loads.



One-Line Diagram of a Balanced Three - Bus Power Grid

Positive Sequence Network Model for Balanced Fault Studies



## 4.5.7 Balanced three-phase faults calculations

- In the previous figure, the load is represented by its equivalent impedance model

$$Z_{load} = \frac{V_{load}^2}{P_{load} - jQ_{load}}$$

- In the design of power grids, the voltage calculation and power flow studies are performed before the short-circuit currents are calculated. Then, the calculated bus load voltages are used to determine the circuit breakers' interrupting capacities. Therefore, the pre-fault voltages are calculated from power flow studies and are known.

$$E_{Bus(0)} = Z_{Bus} I_{Bus(0)}$$

- $E_{Bus(0)}$  is the bus voltage vector before the fault and  $Z_{Bus}$  is the bus impedance matrix model with respect to the ground bus.  $I_{Bus(0)}$  is the generator injected currents before the fault.

## 4.5.7 Balanced three-phase faults calculations

- During the fault, the faulted network variables are designated by “F” and the bus voltage during a fault can be expressed as

$$E_{Bus(F)} = E_{Bus(0)} - Z_{Bus(F)} I_{Bus(F)}$$

- Where  $E_{Bus(F)}$  is the bus voltage vector during the fault,  $E_{Bus(0)}$  is the bus voltage before the fault, and  $Z_{Bus(F)}$  is the bus impedance matrix, and  $I_{Bus(F)}$  is the bus fault current during the fault. For the bus system of previous Fig. with a fault at bus 3, we have:

$$\begin{bmatrix} E_{1(F)} \\ E_{2(F)} \\ E_{3(F)} \end{bmatrix} = \begin{bmatrix} E_{1(0)} \\ E_{2(0)} \\ E_{3(0)} \end{bmatrix} - \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ I_{3(F)} \end{bmatrix}$$

- $E_{1(F)} = E_{1(0)} - Z_{13} I_{3(F)}$      $E_{2(F)} = E_{2(0)} - Z_{23} I_{3(F)}$      $E_{3(F)} = E_{3(0)} - Z_{33} I_{3(F)}$  (4.11)
-

## 4.5.7 Balanced three-phase faults calculations

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- If the fault has impedance  $Z_f$ , then the voltage across the fault impedance is given as

$$E_{3(F)} = Z_f I_{3(F)} \quad (4.12)$$

- From 4.11 and 4.12 we get  $Z_f I_{3(F)} = E_{3(0)} - Z_{33} I_{3(F)}$

- Therefore, the fault current at bus 3 for a balanced three - phase fault can be calculated as

$$I_{3(F)} = \frac{E_{3(0)}}{Z_{33} + Z_f} \quad (4.13)$$

- In Equation 4.13,  $E_{3(0)}$  is pre - fault voltage and  $Z_{33}$  is the Thevenin impedance of bus 3 with respect to the ground bus.
- For the general case with a balanced three - phase fault on a bus “i”, we have:

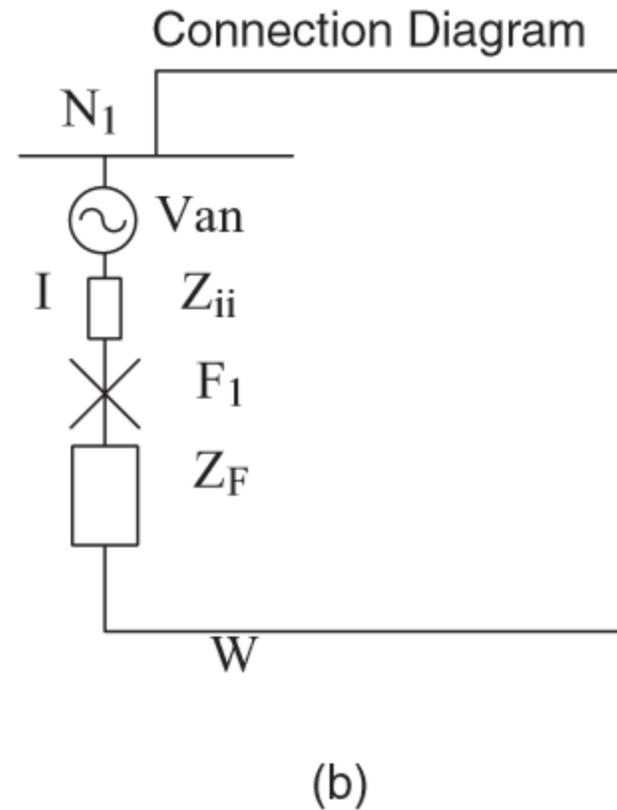
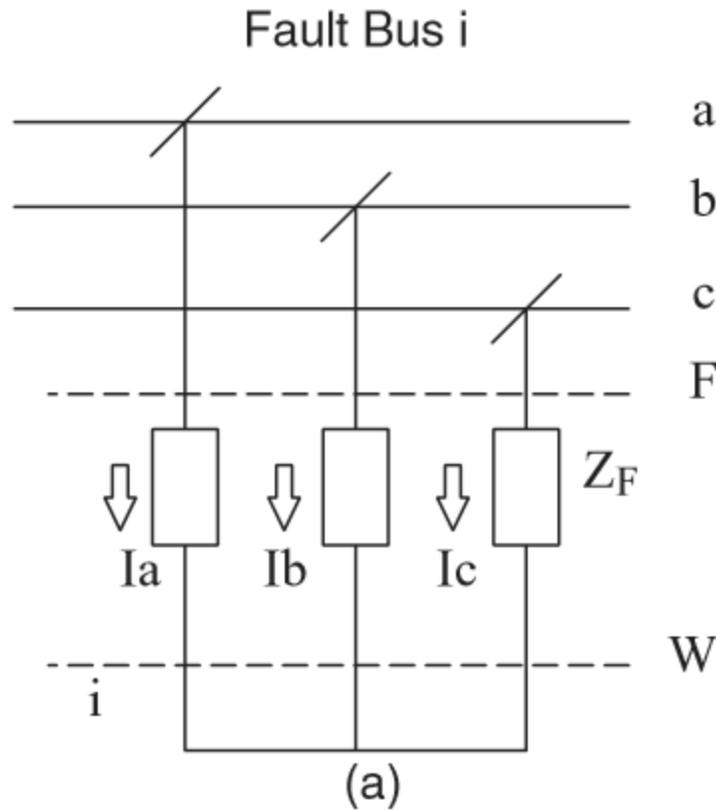
## 4.5.7 Balanced three-phase faults calculations

$$\begin{bmatrix} E_{1(F)} \\ E_{2(F)} \\ \cdot \\ \cdot \\ E_{i(F)} \\ \cdot \\ \cdot \\ E_{n(F)} \end{bmatrix} = \begin{bmatrix} E_{1(0)} \\ E_{2(0)} \\ \cdot \\ \cdot \\ E_{i(0)} \\ \cdot \\ \cdot \\ E_{n(0)} \end{bmatrix} - \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1i} & \dots & Z_{1n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ Z_{i1} & Z_{i2} & \dots & Z_{ii} & \dots & Z_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ Z_{n1} & Z_{n2} & \dots & Z_{nP} & \dots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ I_{i(F)} \\ 0 \\ \cdot \\ 0 \end{bmatrix}$$

- From the above, we can express the fault at bus  $i$  as:  $E_{i(F)} = E_{i(0)} - Z_{ii} I_{P(F)}$

- Therefore, the fault current at bus  $i$  is  $I_{i(F)} = \frac{E_{i(0)}}{Z_{ii} + Z_f}$

## 4.5.7 Balanced three-phase faults calculations



The Balanced Three -  
Phase (a) Fault, and (b)  
the Thevenin Equivalent  
Circuit.

## 4.5.7 Balanced three-phase faults calculations

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### Example 4

Consider a microgrid as part of an interconnected power grid. Assume the following data:

- Local power grid short-circuit capacity=320 MVA
- PV - generating station #1: PV arrays = 2MVA, internal impedance = highly resistive, 50% of its rating
- Gas turbine station: combined heat and power (CHP) units = 10 MVA, internal reactance = 4 %. Units are Y connected and grounded.
- Transformers = 460 VY grounded/13.2 kV  $\Delta$  , 10% reactance, 10 MVA capacity
- Power grid transformer: 20 MVA, 63 kV/13.2 kV, 7% reactance
- Bus 4 load = 1.5 MW, power factor (p.f.) = 0.85 lagging; bus 5 load = 5.5 MW, p.f. = 0.9 lagging; bus 6 load = 4.0 MW, p.f. = 0.95 leading; bus 7 load = 5 MW, p.f. = 0.95 lagging; bus 8 load = 1.0 MW, p.f. = 0.9 lagging

## 4.5.7 Balanced three-phase faults calculations

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### Example 4

Transmission line: resistance =  $0.0685 \Omega$  /mile, reactance =  $0.40 \Omega$  /mile, and half of line charging admittance ( $Y' / 2$ ) of  $11 \times 10^{-6} \Omega^{-1}$ /mile. Line 4 – 7 = 5 miles, 4 – 8 = 1 mile, 5 – 6 = 3 miles, 5 – 7 = 2 miles, 6 – 7 = 2 miles, 6 – 8 = 4 miles

- Perform the following:
  - i. Develop a per unit equivalent model for balanced three - phase fault studies based on a 20 M VA base
  - ii) If the bus load voltages are at 1 per unit, compute the per impedance model of each load.
  - iii) For three - phase faults, compute the SCC (short circuit current) of each distribution network bus
  - iv) To increase the security of the system two identical transformers are used at each distribution network and the interconnection to the local power grid. Compute the SCC of each bus.

## 4.5.7 Balanced three-phase faults calculations

i) The base value of the volt-amp is selected as  $S_b = 20$  MVA. The voltage base selected on the PV generator and gas turbine side is selected to be 460 V. The voltage base on the transmission lines side is therefore  $V_b = 13.2$  kV

- The p.u SCC of the local power grid is given by  $\frac{SCC}{S_b} = \frac{320}{20} = 16$

- Therefore, the internal p.u impedance of the local power grid is

$$Z_{th} = \frac{1}{SCC_{p.u}} = \frac{1}{16} = j0.063$$

- The internal p.u impedance of the PV - generating station at 20 MVA base is given by

$$\begin{aligned} Z_{p.u(new)} &= Z_{p.u(old)} \times \frac{VA_{b(new)}}{VA_{b(old)}} \times \left( \frac{V_{b(old)}}{V_{b(new)}} \right)^2 \\ &= 0.5 \times \frac{20 \times 10^6}{2 \times 10^6} \times \left( \frac{460}{460} \right)^2 = 5 \end{aligned}$$

## 4.5.7 Balanced three-phase faults calculations

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- The p.u impedance of the gas turbine is

- $$z = j0.4 \times \frac{20 \times 10^6}{10 \times 10^6} \times \left( \frac{460}{460} \right)^2 = j0.8$$

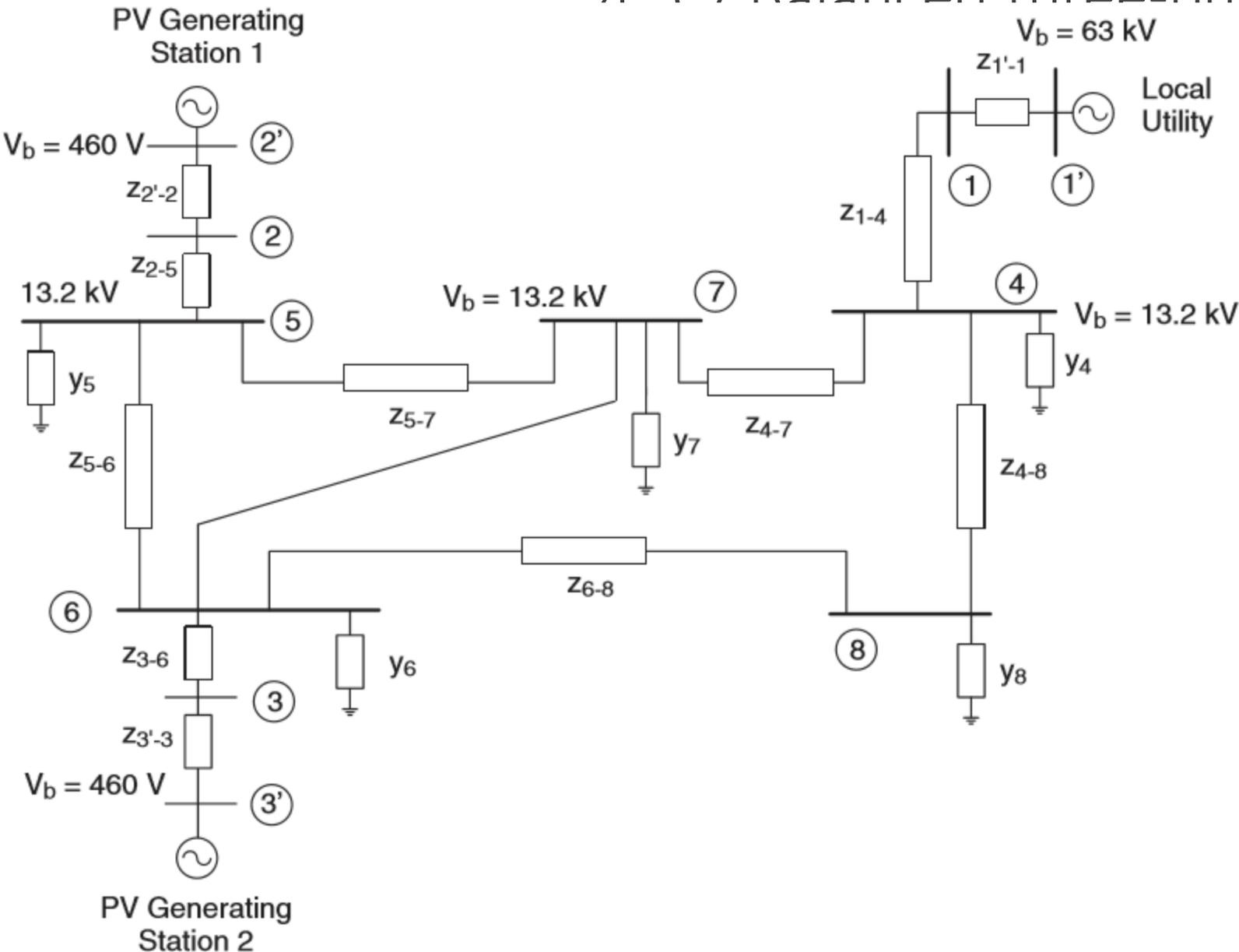
- The base impedance of the transmission system is

$$Z_b = \frac{V_b^2}{S_b} = \frac{(13.2 \times 10^3)^2}{20 \times 10^6} = 8.712 \Omega$$

- The base admittance given by

$$Y_b = \frac{1}{Z_b} = \frac{1}{8.712} = 0.115$$

# 1 5 7 Balanced three-phase faults calculations



Impedance Model for Short - Circuit Studies

## 4.5.7 Balanced three-phase faults calculations

- The p.u impedance of the local power grid transformer is 7% based on 20 MVA.
- The loads represented by their equivalent impedance are calculated from the equation

$$z_{load} = \frac{V_{load}^2}{P_{load} - jQ_{load}}$$

- The per unit impedance of a line between bus  $i$  and  $j$  is given by

$$z_{i-j,p.u} = \frac{z_{i-j}}{Z_b}$$

- Using the above equation, the transmission line p.u parameters are calculated and listed in the next Table:

Line	Series Impedance (p.u)
1-4	$j0.07$
2-5	$j0.2$
3-6	$j0.2$

## 4.5.7 Balanced three-phase faults calculations

Line	Series Impedance (p.u)	Line Charging Admittance (p.u)
4-7	$0.039 + j0.229$	$j479 \times 10^{-6}$
4-8	$0.008 + j0.046$	$j96 \times 10^{-6}$
5-6	$0.024 + j0.138$	$j287 \times 10^{-6}$
5-7	$0.016 + j0.092$	$j192 \times 10^{-6}$
6-7	$0.016 + j0.092$	$j192 \times 10^{-6}$
6-8	$0.031 + j0.184$	$j383 \times 10^{-6}$

Bus	Load	Complex Power (p.u)	Equivalent Load Impedance (p.u)
4	1.5 MW at 0.85 p.f. (lagging) $= 1.5 + j0.92$	$0.075 + j0.046$	$9.69 + j5.94$
5	5.5 MW at 0.9 p.f. (lagging) $= 5.5 + j2.66$	$0.275 + j0.133$	$2.95 + j1.43$
6	4.0 MW at 0.95 p.f. (leading) $= 4.0 - j1.31$	$0.20 - j0.066$	$4.51 - j1.49$
7	5 MW at 0.95 p.f. (lagging) $= 5.0 + j1.64$	$0.25 + j0.082$	$3.61 + j1.18$
8	1.0 MW at 0.9 p.f. (lagging) $1.0 + j0.48$	$0.05 + j0.024$	$16.25 + j7.80$

Transmission Line Parameters

The load of each Bus and its equivalent Load Impedance

## 4.5.7 Balanced three-phase faults calculations

- The elements of YBus matrix can be calculated from the following algorithm:
- Step 1. If  $i=j$ ,  $Y_{ij}=\Sigma y$ .
- Step 2. If  $i\neq j$  and bus  $i$  is not connected to bus  $j$  then  $Y_{ij}=0$
- Step 3. If  $i\neq j$  and bus  $i$  is connected to bus  $j$  through the admittance  $y_{ij}$  then  $Y_{ij}=- y_{ij}$
- For short-circuit studies, it is industry practice to omit the line charging and the load impedances for calculating the balanced fault current for each bus. However, we can include the load impedance by using the bus load voltages from a power flow calculation.

$$Z_{load} = \frac{V_{load}}{I_{load}} = \frac{V_{load}}{(V_{load} / I_{load})^*} = \frac{V_{load}^2}{S_{load}^*}$$

## 4.5.7 Balanced three-phase faults calculations

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- However, because in a designed power grid the load bus voltages are around 1 per unit with a tolerance of 5%, we can use 1 per unit for load voltages and compute the load impedance for use in short-circuit studies.
- The  $Y_{\text{Bus}}$  matrix for short-circuit studies will include the internal impedance of the generator buses. For this exercise, the  $Y_{\text{Bus}}$  matrix will be  $8 \times 8$  with the voltage sources replaced by their internal impedance to find the Thevenin's equivalent impedance.



## 4.5.7 Balanced three-phase faults calculations

$$SCC_{p.u} = \frac{1}{Z_{ii}} \quad i = 1, 2, \dots, 8$$

- where  $Z_{ij}$  is the diagonal element of  $Z_{BUS}$  matrix. The SCC of each bus and the SCC of each bus when two transformers are used for each generation bus is given in Tables below

Bus	SCC (p.u)	Bus	SCC (p.u)
1	16.71	1	16.81
2	2.21	2	3.05
3	3.42	3	4.31
4	8.41	4	11.18
5	3.92	5	4.36
6	4.82	6	5.51
7	4.57	7	5.20
8	6.45	8	7.91

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