Lecture 3: Performance analysis tools for Smart Grid design

George Konstantinou, University of Cyprus

01/08/2020

Nicosia, Cyprus

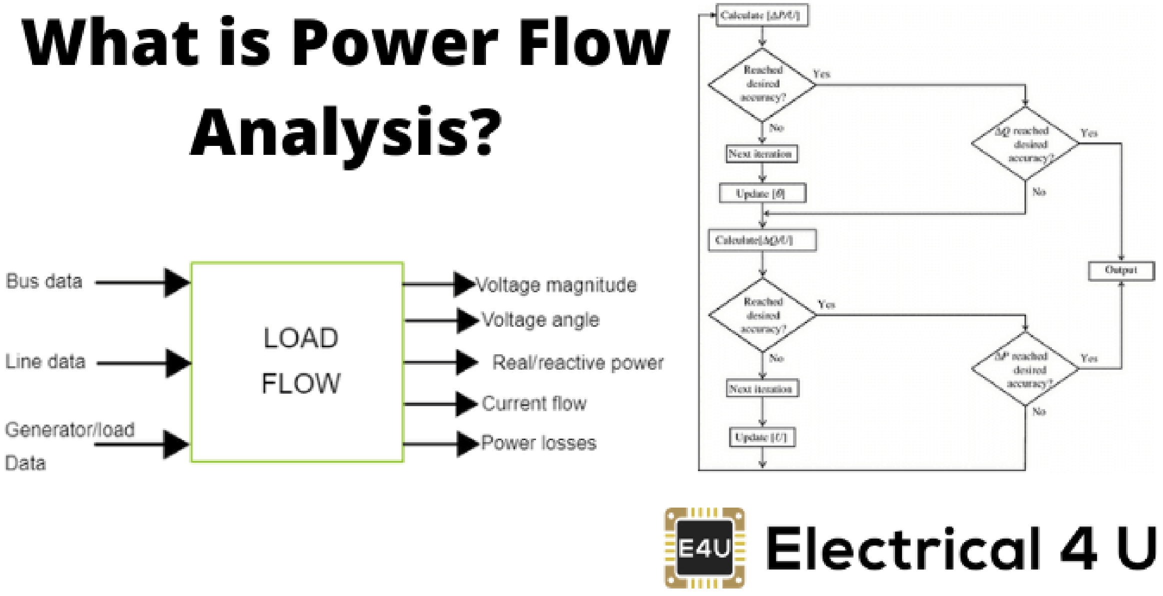
Course material developed in collaboration with Technical University of Sofia, University of Western Macedonia, International Hellenic University, University of Cyprus, Public Power Corporation S.A., K3Y Ltd and Software Company EOOD

with support from Erasmus +

Content of the lecture

3.1 Introduction to load flow studies

- Load flow studies are critical to system planning and system operation.
- Load flow studies identify line loads and bus voltages out of range, inappropriately large bus phase angles (and the potential for stability problems), component loads (principally transformers), proximity to Q-limits at generation buses, and other parameters having the potential to create operating difficulties.
- Load flow studies assist system operators in calculating power levels at each generating unit for economic dispatch, analyzing outages and other forced operating conditions, and coordinating power pools.



3.2 Challenges to Load Flow in Smart Grid

- **Four important questions should be answered:**

1. What are the special features of the smart grid compared to the legacy system?
2 . What computations are needed in the case of smart grid?
3. What specific directions are needed for developing a new power flow?
4. What new features of the load flow make it suitable for smart grid performance and evaluation?

- ***Other features to be considered in the development of the new load flow include:***

1. Condition adaptiveness of transmission and distribution to accommodate load flows comprising renewable generation
2. Self - adaptiveness to ensure proper coordination
3. High impedance topology matching for distribution network with randomness and uncertainty requiring intelligence analytical tools
4. Since reverse power flow technique is possible, the use of FACTS (Flexible AC Transmission Systems) devices to power electronics building blocks is essential.

3.2 Challenges to Load Flow in Smart Grid

Old Load Flow Technique	Desired Load Flow Technique
Central generation and control	Central and distributed generation control and distributed intelligence
Load flow by Kirchhoff's laws	Load flow by electronics
Power generation according to demand	Controllable generation, fluctuating/random sources and demand in dynamic [1,6] equilibrium
Manual switching and trouble response	Automatic response and predictive avoidance
Simulation and response tracking	Monitoring overload against bottlenecks

Load flow techniques comparison

3.3 Load Flow State of the Art: Classical, Extended Formulations, and Algorithm

- The traditional load flow techniques used for distribution load flow are characterized by:

1. Distribution systems are radial or weakly meshed network structures
2. High X/R ratios in the line impedances (this is typical of higher voltage lines that the phases are spaced further appart).
3. Single phase loads handled by the distribution load flow program
4. Distributed Generation (DG), other renewable generation, and/or cogeneration power supplies installed in relative proximity to some load centers
5. Distribution systems with many short line segments, most of which have low impedance values

3.3 Load Flow State of the Art: Classical, Extended Formulations, and Algorithm

- For the purpose of load flow study we model the network of buses connected by lines or switches connected to a voltage - specific source bus. The load and/or generator are connected to the buses.
- The classical methods of studying load flow include:

1. Gauss – Seidal
2. Newton – Raphson
3. Fast Decouple

3.3.1 Gauss–Seidal Method

- This method uses Kirchhoff 's current law nodal equations given as $I_{injection}$ current at the node. Suppose $I_{injection}$ = current at the node of a given connected load, then

$$I_{inj(j)} = \sum_{i=1}^{n} I_{ji}$$

- where I_{inj} *(j)* is the injection current at bus *j* and I_{ji} = current flow from jth bus to ith bus. Rewriting, we obtain I_{inj} *(j)* = *Y*bus*V*bus where *Y*bus admittance matrix is given as *V*bus vector of bus voltages.
- If we sum the total power at a bus, the generation and load is denoted as complex power. The nonlinear load flow equation is $S_{inj-k} = P_g + jQ_g - (P_{LD} + jQ_{LD})$

$$= V_k \left(\sum_{j=1}^{n} Y_{kj} V_j \right)^*$$

3.3.1 Gauss–Seidal Method

- This equation is solved by an iterative method for V_j if P and Q are specified. Additionally, from

$$V_i^{(k+1)} = g(V_{\text{bus}}^k)$$
$$= \frac{1}{Y_{ii}}\left(\frac{P_L^{sch} - jQ_L^{sch}}{V_L^{*(k)}} - \sum_{j=1}^{n} Y_{ij}V_j^{(k)}\right)$$

- where Y_{ij} are the elements of bus admittance matrix, and P_i^{sch} and Q_i^{sch} are scheduled P and Q at each bus.
- After a node voltage is updated within iteration, the new value is made available for the remaining equations within that iteration and also for the subsequent iteration. Given that the initial starting values for voltages are close to the unknown, the iterative process converges linearly.

3.3.2 Newton–Raphson Method

- The Newton – Raphson Method assumes an initial starting voltage use in computing mismatch power ΔS where

$$\Delta S = S_{ij-i}^{sch} - (V_i^{|k|})^* (\sum Y_{ij} V_j^k)$$

- The expression ΔS is called the mismatch power. In order, to determine convergence criteria given by $\Delta S \leq \varepsilon$, where ε is a specific tolerance, accuracy index, and a sensitivity matrix is derived from the inverse Jacobian matrix of the injected power equations:

$$P_i = |V_i| \sum |Y_{ij}| |V_j| \cos(\theta_i - \theta_j - \psi_{ij})$$

$$Q_i = |V_i| \sum |Y_{ij}| |V_j| \sin(\theta_i - \theta_j - \psi_{ij})$$

where ϑ_i is the angle between V_i and V_j, and ψ_{ij} is the admittance angle.

- The complex power ΔS can be expressed in polar or rectangular form

$$\Delta V = |\Delta V| \angle \theta_v \quad \text{or} \quad \Delta S = \Delta P + \Delta Q$$

respectively.

- This method is excellent for large systems but does not take advantage of the radial structure of distribution and hence is computationally inefficient. The method fails when the Jacobian matrix is singular or the system becomes ill - conditioned as in the case of a low distribution X/R ratio.

3.3.3 Fast decouple method

- The fast decouple method simplifies the Jacobian matrix by using small angle approximations to eliminate relatively small elements of the Jacobian. The method is one of the effective techniques used in power system analysis. However, it exhibits poor convergence with a high R/X ratio system.
- The interaction of V and θ magnitudes with active and reactive power flows cause poor convergence as well. A variation of this method solves current injection instead of model power injection power equations.

Method 1: Forward/Backward Sweep. This method models the distribution system as a tree network, with the slack bus denoted as the root of the tree and the branch networks as the layers which are far away from the root nodal. Weakly meshed networks are converted to a radial network by breaking the loops and injection currents computation.

The backward sweep primarily sums either the line currents or load flows from the extreme feeder (leaf) to the root. The steps of the algorithm are:

1. Select the slack bus and assume initial voltage and angle at the root, node, and other buses
2. Compute nodal current injection at the *Kth* iteration: $I_i^{(k)} = \left[\frac{S_i^{sch}}{V_i^{(k-1)}} \right]^*$

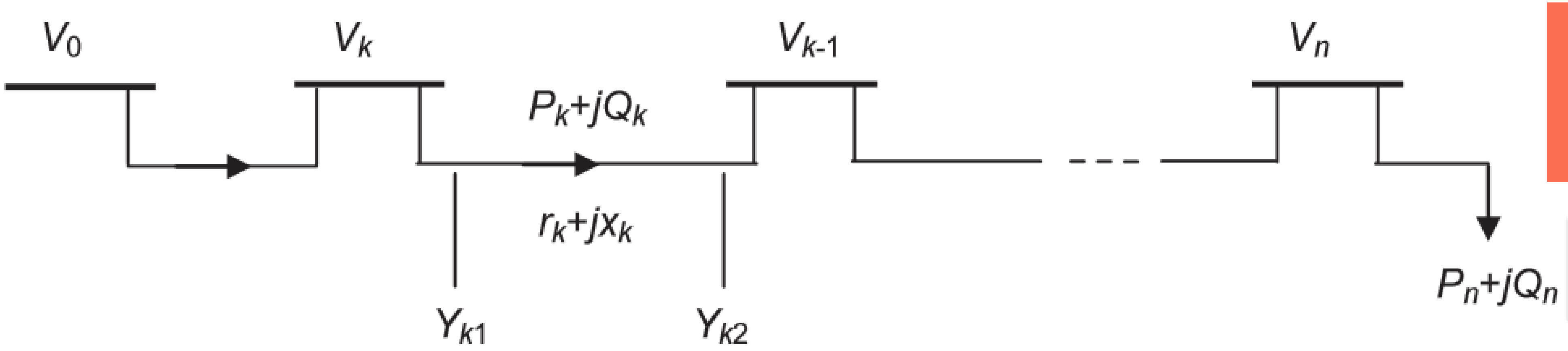
3. Start from the root with known slack bus voltages and move toward the feeder and lateral ends
4. Compute the voltage at node *j*: $V_j^{k-1} = V_i^k - Z_{ij} I_{ij}^{(k)}$

where Z_{ij} *is the branch impedance between bus* i *and* j *and* V_j *is the latest voltage value of bus j.*

5. Compute the power mismatch from and check the termination criteria using

$\Delta S_i^{(k)} = S_i^{sch} - V_i^{(k)} \left(I_i^{(k)}\right)^* \leq \varepsilon$ *as before for all interconnected branches.*

Method 2: Load Flow Based on Sensitivity Matrix for Mismatch Calculation. The distribution load flow is an improved forward/backward method utilizing a sensitivity matrix scheme to compensate the mismatch between slack bus power injection and the load flow at the feeder and lateral ends. This results in the Newton – Raphson method for distribution load flow.



Single feeder representation

- Consider a single feeder, as shown in the previous figure. The steps for this method are:

1. Assume the slack bus as the root node
2. Assume P_0 , Q_0 power injection at the slack bus node equal to the sum of all of the loads in the system
3. Load flows in each branch are equal to the sum of downstream connected loads. At *k*th iteration, start from root node with known voltage at slack bus
4. Obtain the latest V^k , P_{ij}^k, Q_{ij}^k (voltage and flows)
5. Compute power loss = $f(V^k, P_{ij}^*, Q_{ij}^*)$
6. From the loss compute receiving and power P_{ji} , Q_{ji}, and V_j
7. The loads and shunt (lost) power are taken from the received power and the remaining power is sent to the next feeder at lateral branches

8. At network solutions ΔP_L, ΔQ_L = 0, when mismatch power is approximately 0; if load flow mismatch is less than the tolerance, ε, then load flow has converged.
9. Update the slack bus power from the sensitivity matrix.

Method 3: Bus Impedance Network. This method uses the bus impedance matrix and equivalent current injection to solve the network equation in a distribution system. It employs a simple superposition to find the bus voltage through the system. The voltage in each bus is computed after specifying the slack bus voltage and then computing the incremental change ΔV due to current injection flowing into the network.

The steps are:

1. Assume no load system
2. Initialize the load bus voltage throughout the system using the value of the slack bus voltage
3. Modify nodal voltages due to current flow which are function of the connected loads
4. The injection current is modified in the K^{th} iteration as level changes

5. Use $I_i^{(k)} = (S_i^{sch} / V_i^{k-1})^*$ for the first equivalent current injection until getting I_j^k at the last iteration I_0^k
6. Compute the vector of voltage denoted as ΔV using $\Delta V_{bus}^{(k)} = Z_{bus} I_{inj}^{(k)}$, , where Z_{bus} is a $\eta \times \eta$ bus impedance matrix
7. Determine the bus voltage updates throughout the network as $V_i^{k-1} = V_0 - \Delta V_i^{(k-1)}$ where V_0 is slack bus voltage at root node
8. Check mismatch power at each load bus using specified and calculated values to obtain

$$\Delta S = S^{spec} - \sum V_i^{calc} I_{ij}^{calc}$$

and stop if the value of $\Delta S \leq \varepsilon$.

Otherwise, go back to step 3.

3.3.4 Distribution Load Flow Methods

- ATC (Available Transfer Capability) is the maximum amount of additional MW transfer possible between two parts of a power system. Additional means that existing transfers are considered part of the base case and are not included in the ATC number.
- Typically, these two parts are control areas or any group of power injections. Maximum refers to cases of either no overloads occurring in the system as the transfer is increased or no overloads occurring in the system during contingencies as the transfer is increased in online real time. By definition, ATC is computed using the formula

$$ATC = TTC - \sum(\text{CBM, TRM, and “existing TC”})$$

where the components are Total Transfer Capability (TTC), Capacity Benefit Margin (CBM), Transmission Reliability Margin (TRM), and existing Transmission Commitments.

- ATC is particularly important because it signals the point where power system reliability meets electricity market efficiency.
- It can have a huge impact on market outcomes and system reliability, so the results of ATC tend to be of great interest to all players.
- The load flow is solved using the iterate solution on

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

to obtain the incremental changes in the LHS vector until the terminating conditions of the power mismatch (or maximum number of iterations) is reached.

- After convergence, derived quantities such as losses and power factors can be computed using network equations.

3.3.4 Distribution Load Flow Methods

- A smart decoupled load flow equation could be developed based on the following assumptions.

Let $\delta_k - \delta_m \approx 0$ such that $\cos(\delta_k - \delta_m) \approx 1$ and $\sin(\delta_k - \delta_m) \approx 0$

$$V_k \approx 1$$

which implies $g_{km} \approx 0$, this allows a reduction of the Jacobian such that

$r_{km} << x_{km}$;

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}$$

can be solved without using an iterative method to solve simultaneous equations. The approximation takes advantage of the lower sensitivities of real power with respect to (w.r.t.) voltage magnitude and reactive power w.r.t. voltage arguments.

- The DC load flow equations are simply the real part of the decoupled load flow equations. This is achieved via the following additional assumptions: only angles and real power in real time are solved for by iterating

$$\Delta\boldsymbol{\delta} = [\mathbf{B}']^{-1}\,\Delta\mathbf{P} \text{ where } \frac{\partial \mathbf{P}}{\partial'} = \mathbf{B}' \text{ with } \frac{\partial P_k}{\partial \delta_k} = \sum_{\substack{m=1 \\ m\neq k}}^{N} b_{km} \text{ and } \frac{\partial P_k}{\partial \delta_m} = -b_{km}.$$

- Recall the load flow that is solved using the iterate on

$$\begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta\mathbf{V} \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix}$$

until the terminating conditions of the power mismatch (or maximum number of iterations) is reached.

3.4 Load Flow for Smart Grid Design

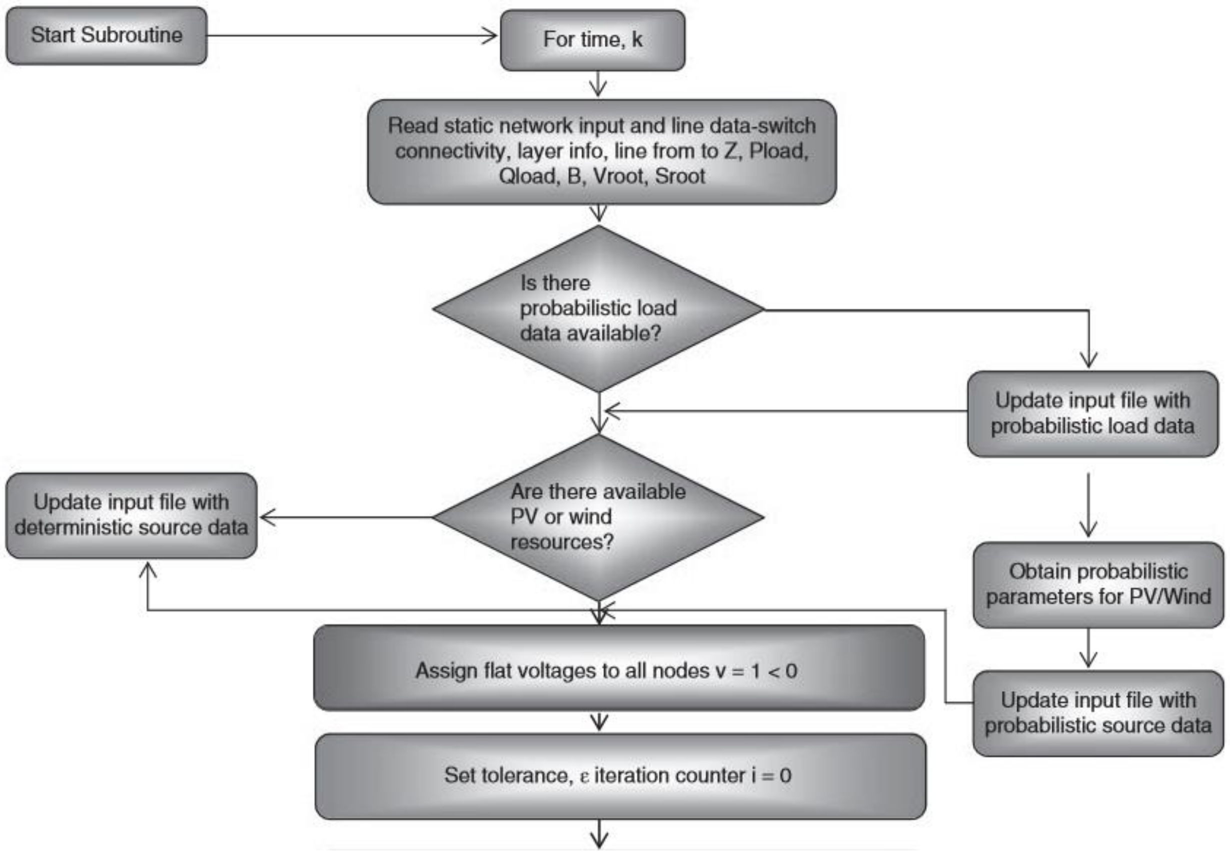
- Load flow tools that incorporate the stochastic and random study of the smart grid could be modeled with an implementation algorithm.
- Conditioning the load flow topology will require a new methodology and algorithm that will include feeders and the evolution of a time-dependent load flow. This method has been proven in terms of characteristics and usage in power system planning and operation.
- Hence, the interoperability of RER (Renewable Energy Resources) with smart grid specifications could account for adequate use of current methodology to perform analysis in both usual and alert states

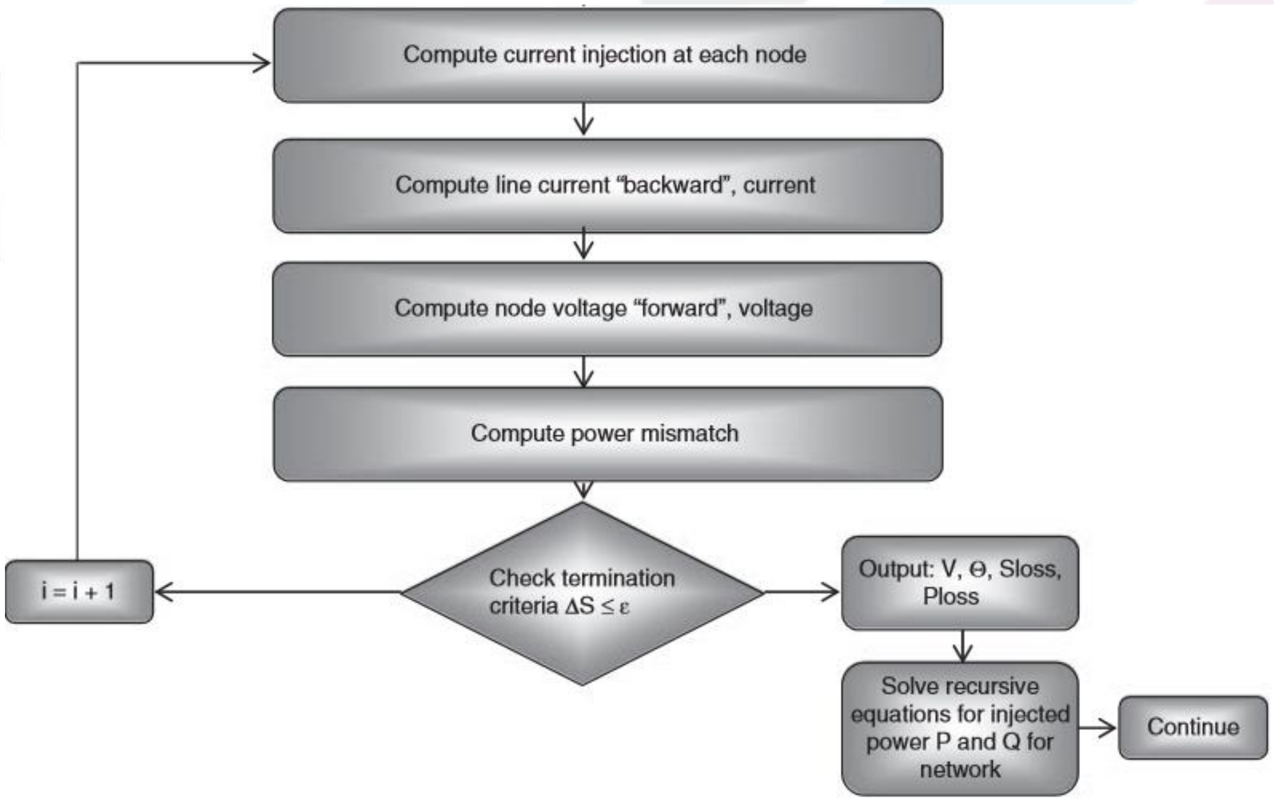
3.4 Load Flow for Smart Grid Design

- The implementation algorithm proposed will extend the following capability:

1. Model input of RER (Renewable Energy Resources) and load will be changed to account for variability; the input will have to include some power distribution flow to advance the congested value of new estimate of P_f, Q_g, and P_d, Q_d. These attributes also have a unique load appropriate effectiveness in the performance study.
2. Sparsity may be affected because the loads of RER may be widely distributed, that is, load and size of RER has to be considered.
3. Computational challenges in new load flow with RER for smart grid that include the stochastic model may affect the independent computation.

3.4 Load Flow for Smart Grid Design

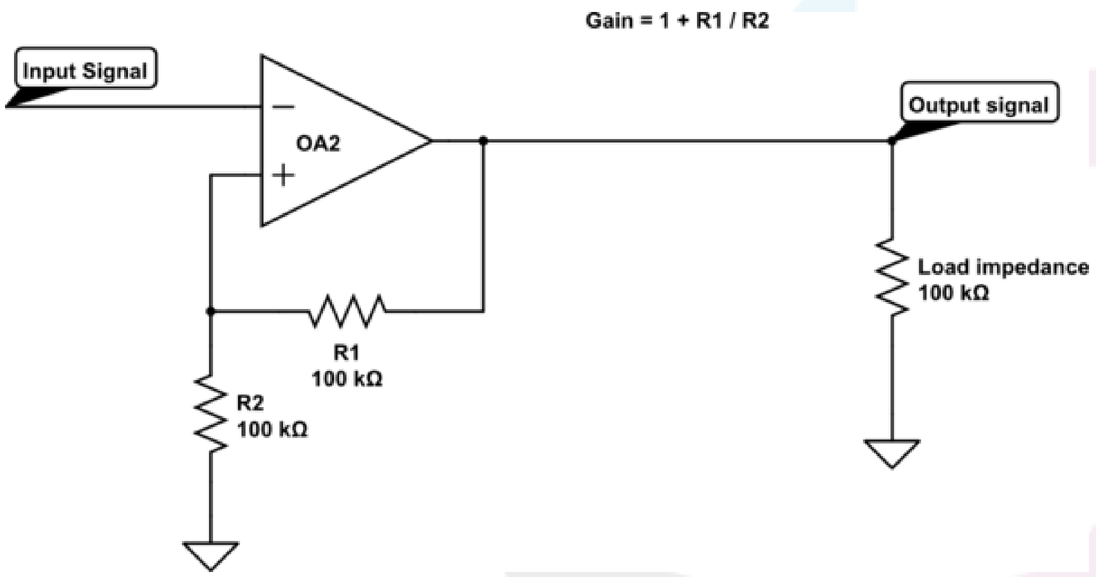




Proposed load flow methodology

3.5 Voltage calculation in power grid analysis

- In a circuit problem, the impedance of loads and the source voltage are given, and the problem is to find the current flow in the circuit and calculate the voltage across each load.
- In voltage calculation in a power problem, the loads are given in terms of active and reactive power consumption.
- We can study this problem via two methods: (1) assume the voltage across the load and calculate the source voltage, and (2) assume the source voltage and compute the bus load voltage (this is known as a power flow or load flow problem).



- We will now illustrate the first method:

Problem 1. A three - phase feeder is connected through two cables with equal impedance of 4 + j15 Ω in series to 2 three-phase loads. The first load is a Y - connected load rated 440 V, 8 KVA, p.f. = 0.9 (lagging) and the second load is a Δ - connected motor load rated 440 V, 6 KVA, p.f. = 0.85 (lagging). The motor requires a load voltage of 440 V at the end of the line on the Δ-connected loads. Perform the following:

i) Give the one - line diagram

ii) Find the required feeder voltage

- **Solution:** The line voltage at bus 3 is equal to 440 V. The rated current drawn by a motor on bus 3 is

$$I_3 = \frac{kVA_{r3}}{\sqrt{3} \cdot V_3} = \frac{6000}{\sqrt{3} \times 440} \angle -\cos^{-1} 0.85 = 7.87 \angle -31.77^\circ \ A$$

- The voltage at bus 2 is given by

$$V_{2,ph} = V_{3,ph} + Z_{2-3} \times I_3 = 440/\sqrt{3} + (4 + j15) \times 7.87 \angle -31.77 = 353.04 \angle 13.7^\circ \ V$$

- The rated current drawn by a load on bus 2 is

$$I_2 = \frac{kVA_{r2}}{3 \cdot V_2} = \frac{8000}{3 \times 353.04} \angle -\cos^{-1} 0.9 = 7.55 \angle -25.84^\circ \ A$$

3.5 Voltage calculation in power grid analysis

- The supply current of the generator is given by

$$I_1 = I_2 + I_3 = 7.55\angle -25.84 + 7.87\angle -31.77 = 15.39\angle -28.87^\circ\ A$$

- The generator phase voltage is given by

$$V_1 = V_2 + Z_{1-2} \times I_1 = 353.04\angle 13.7 + (4 + j15) \times 15.39\angle -28.87 = 569.21\angle 26.73^\circ\ V$$

- The line voltage of the generator is given by

$$V_1\sqrt{3} = 569.21 \times \sqrt{3} = 985.90\ V$$

- The previous example is not a practical problem. The generator voltage is controlled by its excitation system. In practice, the field current is set to obtain the rated generator voltage. If the generator has two poles and the generator is operating at synchronous speed, that is, for a 60- Hz system, it operates at 3600 rpm.

- Therefore, in a practical problem, we need to compute the voltage at load buses given the generator voltage and load power consumption. The solution to this latter problem — known as the power flow problem — is more complex.

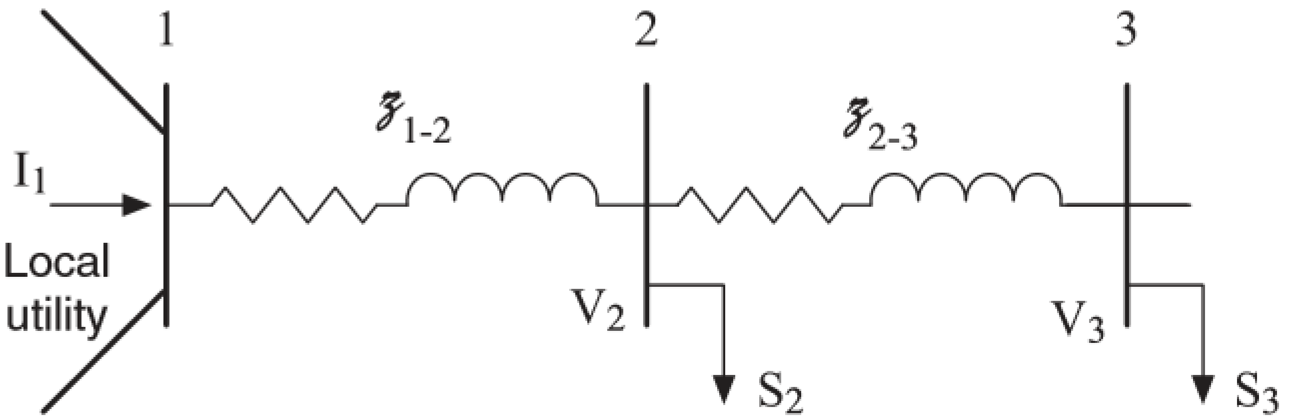
Problem 2. Consider a distributed feeder presented in the next slide Figure. Assume the following:

a. Feeder line impedances, that is Z_{1-2} and Z_{2-3} , are known.

b. The active and reactive power consumed, that is S_2 and S_3, by loads are known.

c. The local power grid bus voltage V_1 is known and all data are in per unit.

3.5 Voltage calculation in power grid analysis



A distribution feeder

Solution: Let us write the Kirchhoff' s current law for each node (bus) and assume that the sum of the currents away from the bus is equal to zero. That is, for buses 1–3, we have:

$$(v_1 - v_2)\,y_{12} - I_1 = 0,$$

$$(v_2 - v_1)\,y_{12} + (v_2 - v_3)\,y_{23} + I_2 = 0$$

$$(v_3 - v_2)\,y_{23} + I_3 = 0$$

where $y_{12} = 1/\ \mathcal{Z}_{1\text{-}2}$ and $y_{23} = 1/\ \mathcal{Z}_{2\text{-}3}$

$$I_1 = \left(\frac{S_1}{V_1}\right)^*, \quad I_2 = \left(\frac{S_2}{V_2}\right)^* \quad \text{and} \quad I_3 = \left(\frac{S_3}{V_3}\right)^*$$

- We may rewrite the system of equations as:

$$\begin{aligned} y_{12}v_1 - y_{12}v_2 &= I_1 \\ -y_{12}v_1 + (y_{12} + y_{23})v_2 - y_{23}v_3 &= -I_2 \\ -y_{23}v_2 + y_{23}v_3 &= -I_3 \end{aligned}$$

Or: $\begin{bmatrix} Y_{11} & Y_{12} & 0 \\ Y_{21} & Y_{22} & Y_{23} \\ 0 & Y_{23} & Y_{33} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ -I_2 \\ -I_3 \end{bmatrix}$ where:

$$Y_{11} = y_{12}, Y_{12} = -y_{12}, Y_{21} = -y_{12}, Y_{22} = y_{12} + y_{23}, Y_{23} = -y_{23}, Y_{32} = -y_{23}, Y_{33} = y_{23}$$

3.5 Voltage calculation in power grid analysis

- The matrix Equation represents the bus admittance matrix.
- The Y_{Bus} matrix is described as $I_{Bus} = Y_{Bus} \cdot V_{Bus}$
- If the system has n buses, I_{Bus} is a vector of n×1 current injection, V_{Bus} is a voltage vector of n×1, and Y_{Bus} is a matrix of nxn.
- Let us continue our discussion for a general case of a power grid with n buses. For each bus, k, we have

$$S_k = V_k I_k^* \qquad k = 1,2,\ldots.n$$

And I_k is the current injection into the power grid at bus k. Therefore, from k row of Y_{Bus} matrix, we have

$$I_k = \sum_{j=1}^{n} Y_{kj} V_j$$

- Substituting this in Power Equation, we have

$$S_k = V_k \left(\sum_{j=1}^{n} Y_{kj} V_j \right)^* \qquad k = 1, 2, \ldots . n$$

- For each bus k , we have a complex equation of the form given by the above equation. Therefore, we have n nonlinear complex equations.

$$P_k = \mathrm{Re}\left\{V_k \sum_{j=1}^{n} Y_{kj}^* V_j^*\right\}$$

$$Q_k = \mathrm{Im}\left\{V_k \sum_{j=1}^{n} Y_{kj}^* V_j^*\right\}$$

$$P_k = V_k \sum_{j=1}^{n} V_j \left(G_{kj} \cos\theta_{kj} + B_{kj} \sin\theta_{kj}\right)$$

$$Q_k = V_k \sum_{j=1}^{n} V_j \left(G_{kj} \sin\theta_{kj} - B_{kj} \cos\theta_{kj}\right)$$

where

$$Y_{kj} = G_{kj} + jB_{kj},\ \theta_{kj} = \theta_k - \theta_j$$

$$V_j = V_j \left(\cos\theta_j + j\sin\theta_j\right)$$

$$V_k = V_k \left(\cos\theta_k + j\sin\theta_k\right)$$

3.5 Voltage calculation in power grid analysis

- For example 2, $n = 3$, we have six nonlinear equations. However, because the power grid bus - voltage magnitude is given and used as a reference with a phase angle of zero, we have four nonlinear equations.
- In example 2, we are given the feeder impedances (Z_{1-2} and Z_{2-3}) and loads (S_2 and S_3). To find the bus load voltages, we need to solve the four nonlinear equations for V2, V3, θ2, and θ3. After calculating the bus voltages, we can calculate the complex power (S1 = P1 + jQ1) injected by the local power feeder. The four nonlinear equations are

$$P_2 = V_2 \sum_{j=1}^{n} V_j (G_{2j} \cos\theta_{2j} + B_{2j} \sin\theta_{2j}) \qquad Q_2 = V_2 \sum_{j=1}^{n} V_j (G_{2j} \sin\theta_{2j} - B_{2j} \cos\theta_{2j})$$

$$P_3 = V_3 \sum_{j=1}^{n} V_j (G_{3j} \cos\theta_{3j} + B_{3j} \sin\theta_{3j}) \qquad Q_3 = V_3 \sum_{j=1}^{n} V_j (G_{3j} \sin\theta_{3j} - B_{3j} \cos\theta_{3j})$$

3.5 Voltage calculation in power grid analysis

- The above expressions present the basic concepts of bus active and reactive power injections of a power grid.
- If we know the bus- injected power, then we can solve for the load bus voltages. Voltage calculation is an important step in the design of a power grid network.
- We should understand that Problem 2 is not the same as Problem 1. Problem 1 is not a realistic problem because we cannot expect the local power grid to provide the voltage at the point of interconnection of a microgrid or a feeder. However, in Problem 2 we know the local power grid bus voltage and our objective is the design of a feeder to provide the rated voltage to its loads.

3.6 Power flow analysis in power grid analysis

- In the design of a power grid, a fundamental problem is the power flow analysis. The solution of the power flow ensures that the designed power grid can deliver adequate electric energy to the power grid loads at acceptable voltage and frequency (acceptable voltage is defined as the rated load voltage).
- For example, for a light bulb rated at 50 W and 120 V, the voltage provided across the load should be 120, with deviation of no more than 5% under normal operating conditions and 10% under emergency operating conditions.
- In per unit (p.u) value, we seek to provide 1 p.u voltage to the loads within the range of 0.95 and 1.05 p.u. That is, once we have specified the schedule of generation to satisfy the system load demand, the solution for bus voltages must be 1 p.u $\pm$ 5%.

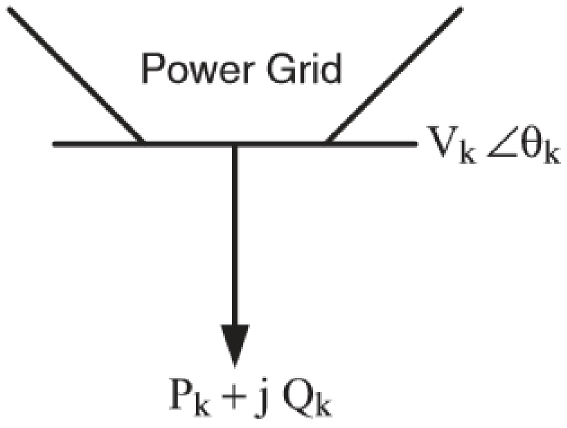
3.6 Power flow analysis in power grid analysis

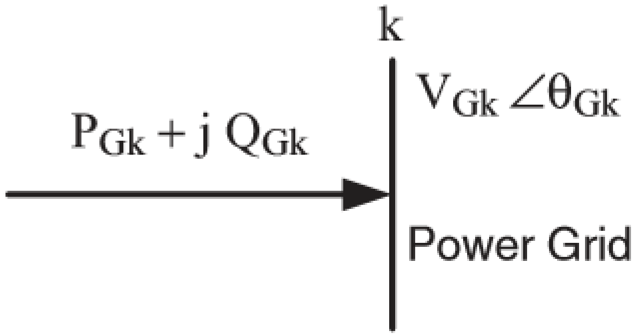
- The acceptable frequency can be ensured if a balance between the system loads and generation is maintained on a second - by - second basis as controlled by load and frequency control and automatic generation control.
- In terms of loads, the frequency deviation from the rated frequency — 60 Hz in the United States and 50 Hz in the rest of the world —affects the machinery loads such as that of the induction motors and pumps.
- If the frequency drops, the speed of the induction motors will drop and result in excessive heating and failure of the induction motors. For example, consider a power system supported by a few diesel generators. If the operating frequency drops due to heavy load demands on the system, the pumps of a diesel generating station slow down; as a result, the diesel generators are not cooled enough. The overheated diesel engines are then removed from service by the system' s override control system, causing a cascade failure of that power system. Therefore, to maintain stable operation, system bus load voltages are maintained at 1 p.u with a deviation of no more than 10% in emergency operating conditions.

3.6.1 Bus types

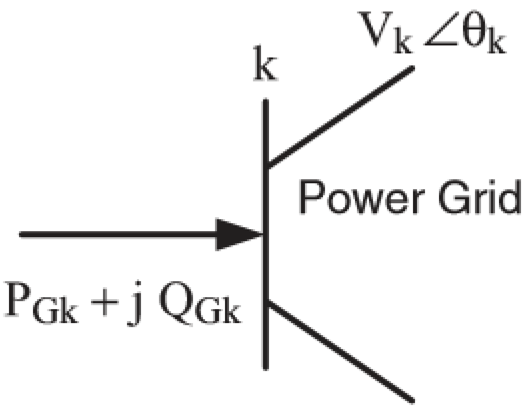
- In a power flow problem, several bus types are defined. The three most important types are a load bus, a generator bus, and a swing bus.
- A power system bus has four variables. These variables are (1) the active power at the bus, (2) the reactive power at the bus, (3) the voltage magnitude, and (4) the phase angle.
- For a load bus, the active and reactive power consumptions are given as a scheduled load for a given time. The time can be specified as the day ahead forecasted peak load. If the system is being planned for 10 years ahead, then the forecasted peak load is used at the bus.

A load bus





A Constant Voltage - Controlled (P-V) Bus (Generator)



A Constant P_G – Q_G Bus

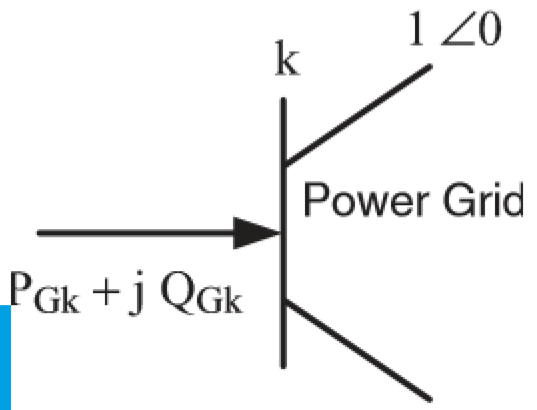
3.6.1 Bus types

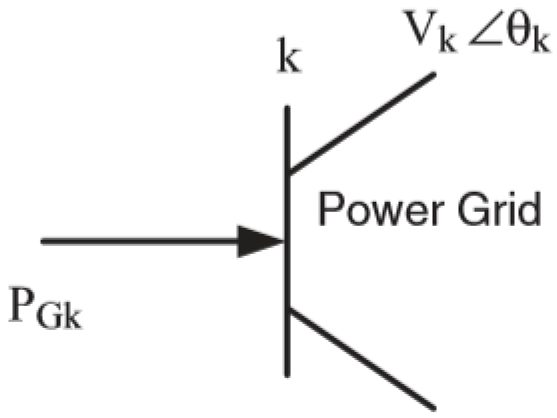
- A P-V (voltage - controlled) bus models a generator bus. For this bus type, the power injected into the bus by the connected generator is given in addition to the magnitude of bus voltage. The reactive power injected into the network and phase angle must be computed from the solution of the power flow problem.
- However, the reactive power must be within the limit (minimum and maximum) of what the P-V bus can provide.
- $P_G - Q_G$ bus. This bus type represents a generator with known active and reactive power injection into the power system. However, the generator magnitude of the voltage and the phase angle must be computed from the solution of power flow problems

3.6.1 Bus types

- The swing bus is identical to a P - V bus except the bus voltage is set to 1 p.u and its phase angle to zero. For a swing bus, the net injected active power and reactive power into the network are not known. The generator connected to the swing bus is called a swing generator or slack generator.
- The function of a swing bus is to balance power consumption and power loss with net - injected generated power. The swing bus can also be considered as an infinite bus, i.e., it can theoretically provide an infinite amount of power. Hence, the swing bus is considered an ideal voltage source: it can provide an infinite amount of power and its voltage remains constant. Note that all the above definitions are identical and are used interchangeably.

Swing bus



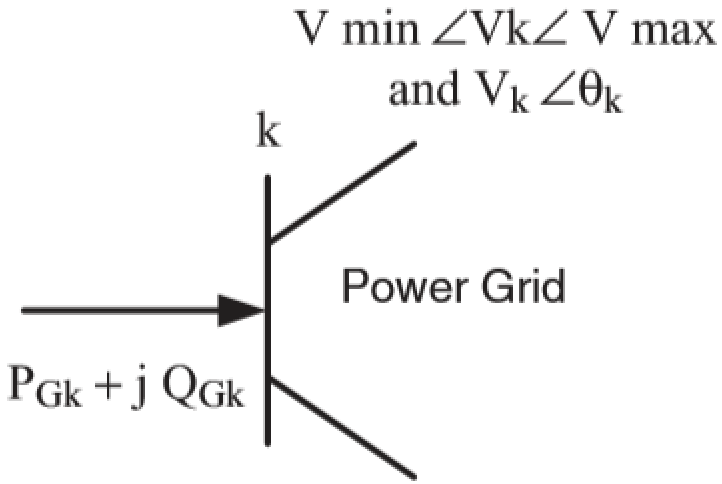


A Photovoltaic or Wind Generating Station Bus Model

- Let us consider the bus type for microgrids of photovoltaic (PV) or wind generating stations. Because the energy captured from the sun or wind source is free, these types of generating units are operated to produce active power. This means that a PV or wind bus is operating at unity power factor (no reactance).

- PV bus depicts the modeling of a PV or wind generating station connected to a bus when a microgrid is connected to the local power grid. In this model, for the voltage analysis of the microgrid, the PV or wind bus active power generation is given and the reactive power generation is assumed to be zero.
- The bus voltage and phase angle is computed from the solution of the power flow problem subject to a minimum and maximum limitation as specified by the modulation index setting of a PV generating station. Therefore, we can summarize the PV generating bus model for bus k as P_{GK} and $V_{min} < V_k < V_{max}$ as specified. At the same time, the reactive power to be provided by a PV or wind generator must be within the limits (minimum and maximum) of the generating station.

- Let us now consider the case when a microgrid is separated from the local power grid. In this case, the local microgrid must control its own frequency and bus voltages. When the microgrid of a PV and wind generating system is separated from the local power grid, the PV or wind generating bus can be modeled as below:



A Photovoltaic or Wind Generating Bus Model

3.6.1 Bus types

- In the above model, the magnitude of bus voltage is specified with a minimum and maximum as defined by the modulation index of the inverter; active power generation and reactive powers are also specified. The phase angle and voltage magnitude are to be computed from the power flow solution.
- However, a PV generating station without a storage system has very limited control over reactive power. To control an inverter power factor, a storage system is essential. To make an inverter with its supporting storage system operate like a steam unit and be able to provide active and reactive power is the subject of ongoing research on modeling and inverter control modeling.
- In case a wind generating station is connected to the microgrid directly, the reactive power injection control is limited within the acceptable voltage range of a connected wind bus.

3.6.1 Bus types

- For an isolated microgrid to operate at a stable frequency and voltage, it must be able to balance its loads and generation at all times.
- Because the load variation is continuous and renewable energy sources are intermittent, it is essential that a storage system and/or a fast - acting generating source such as high - speed microturbines, and/or a combined heat and power generating station be part of the generation mix of the microgrid.
- In a load flow problem, all buses within the network have a designation. In general, the load buses are modeled as a constant P and Q model where the active power, P and reactive power, Q are given and bus voltages are to be calculated. It is assumed that power flowing toward loads is represented as a negative injection into the power system network.
- The generator buses can be modeled as a constant P_G and Q_G or as PV bus type. The generators inject algebraically positive active and reactive power into the power system network. For formulation of the power flow problem, we are interested in injected power into the power system network; the internal impedance of generators is not included in the power system model. However, for short- circuit studies, the internal impedance of the generators is included in the system model.

- The internal impedance limits the fault current flow from the generators. For viable power flow, the balance of the system loads and generation must be maintained at all times.
- This balance can be expressed as:

$$\sum_{k=1}^{n_1} P_{Gk} = \sum_{k=1}^{n_2} P_{Lk} + P_{losses}$$

- where P_{Gk} is the active power generated by generator *k*, P_{Lk} is the active power consumed by the load, n_1 is the number of the system generators, and n_2 is the number of the system loads.

- Similarly,

$$\sum_{k=1}^{n_1} Q_{Gk} = \sum_{k=1}^{n_2} Q_{Lk} + Q_{losses}$$

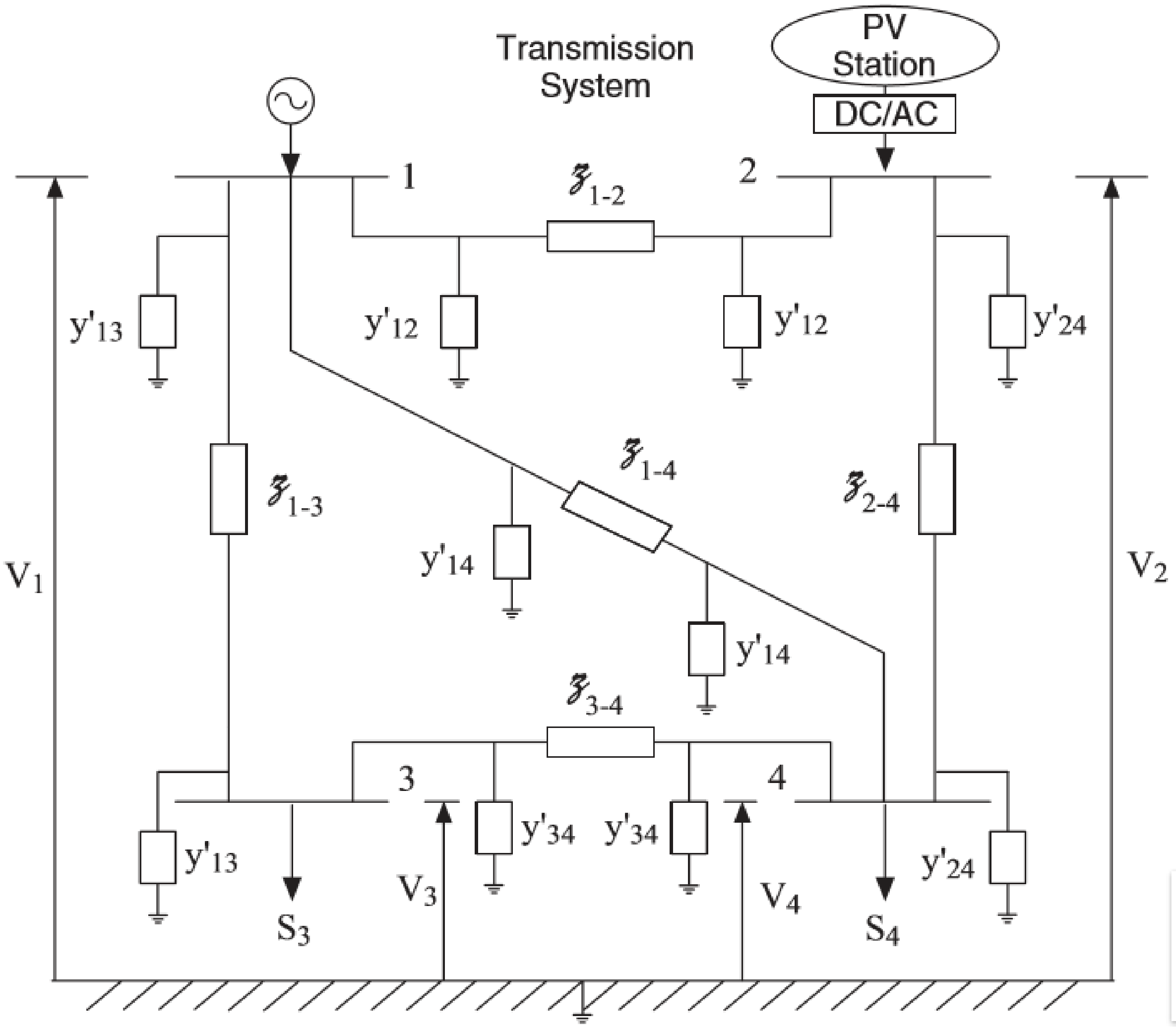
- where Q_{Gk} is the reactive power generated by generator bus k, Q_{Lk} is the reactive power consumed by load k, n_1 is the number of system generators, and n_2 is the number of system loads.

3.6.1 Bus types

- Let us consider the system depicted in Fig. below, in which we must balance the three-bus power loads and generation.

The Schematic Presentation of a Three- Bus Microgrid System

$$P_{G1} + P_{G2} = P_{L4} + P_{L3} + P_{losses}$$
$$P_{G1} + P_{G2} - P_{L4} - P_{L3} - P_{losses} = 0$$
$$Q_{G1} + Q_{G2} = Q_{L4} + Q_{L3} + Q_{losses}$$
$$Q_{G1} + Q_{G2} - Q_{L4} - Q_{L3} - Q_{losses} = 0$$



- In the above formulation, we assume ***inductive loads consume*** reactive power $Q_{Ind} > 0$ and ***capacitive loads supply*** reactive power $Q_{Cap} < 0$.
- To ensure the balance between load and generation, we must calculate the active and reactive power losses.
- However, to calculate power losses, we need the bus voltages. The bus voltages are the unknown values to be calculated from the power flow formulation. This problem is resolved by defining a bus of the power grid — a swing bus and the generator behind it as a swing generator as defined earlier. By definition, the swing bus is an ideal voltage source. As an ideal voltage source, it provides both active and reactive power while the bus voltage remains constant. Therefore, a swing generator is a source of infinite active and reactive power in a power flow problem formulation.
- The swing bus voltage is set to 1 p.u and its phase angle as the reference angle set to zero degree, $V_s = 1\angle 0$. With this assignment, the generator behind the swing bus can provide the required power to the loads of the power grid and its voltage will not be subject to fluctuations.

- Let us now formulate the same problem, for a network of a power grid. We should keep in mind the following assumptions:

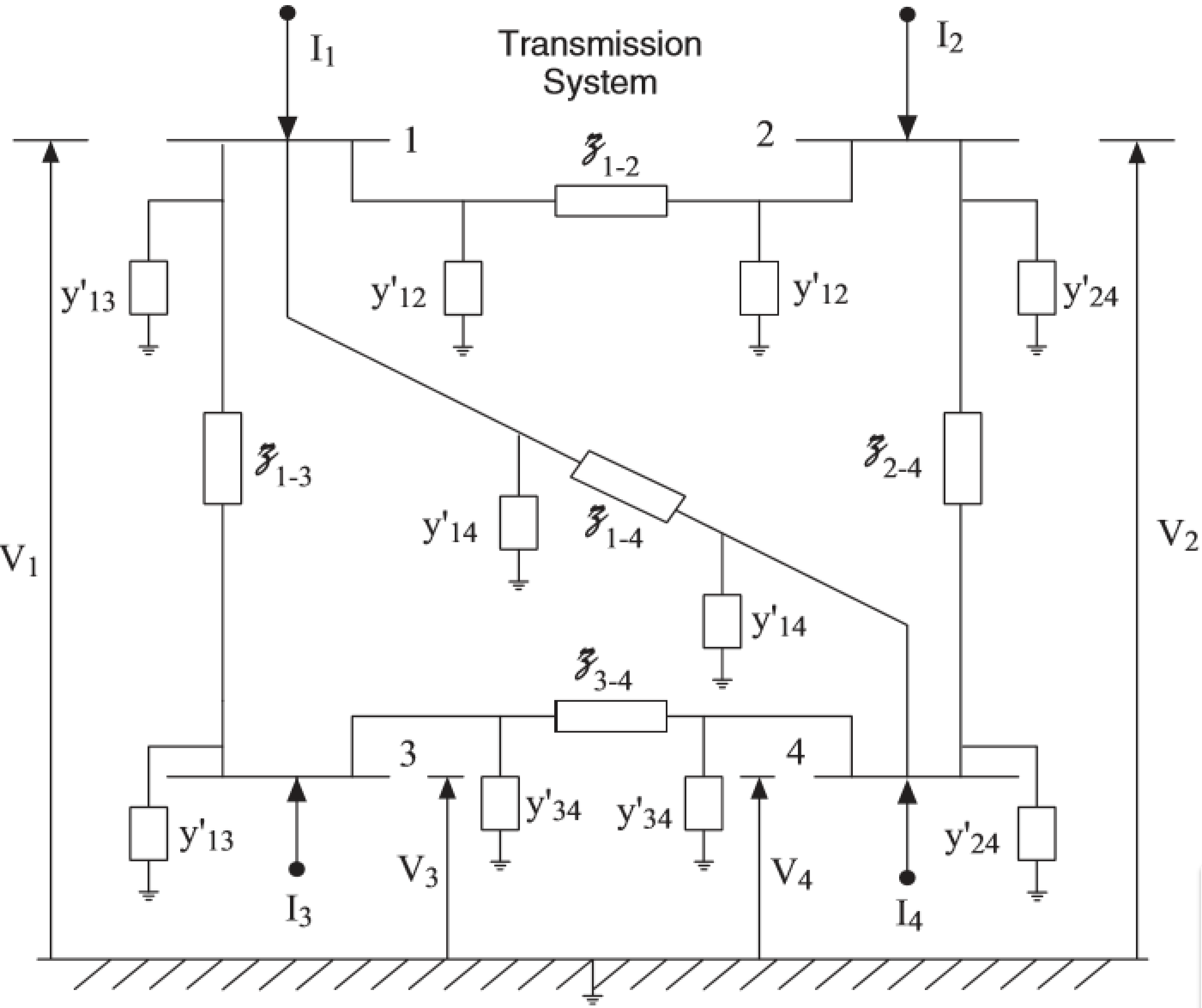
a. The generators are supplying balanced three - phase voltages.

b. The transmission lines are balanced.

c. The loads are assumed to be balanced.

d. The PV or wind generating stations are presented by a PV bus with the bus voltage having a minimum and maximum limit.

- Consider the injection model of a power grid given in the next Fig. The current injections at each bus are presented based on the known power injection and the bus voltage that is calculated from the mathematical model of the system.



A Current Injection Model for Power Flow Studies

The following definitions are implied:

- The bus voltages are actual bus-to-ground voltages in per unit.
- The bus currents are net injected currents in per unit flowing into the transmission system from generators and loads.
- All currents are assigned a positive direction into their respective buses. This means that all generators inject positive currents and all loads inject negative currents.
- The $Z_{i\text{-}j}$ is the one - phase primitive impedance, also called the positive sequence impedance between bus i and j.

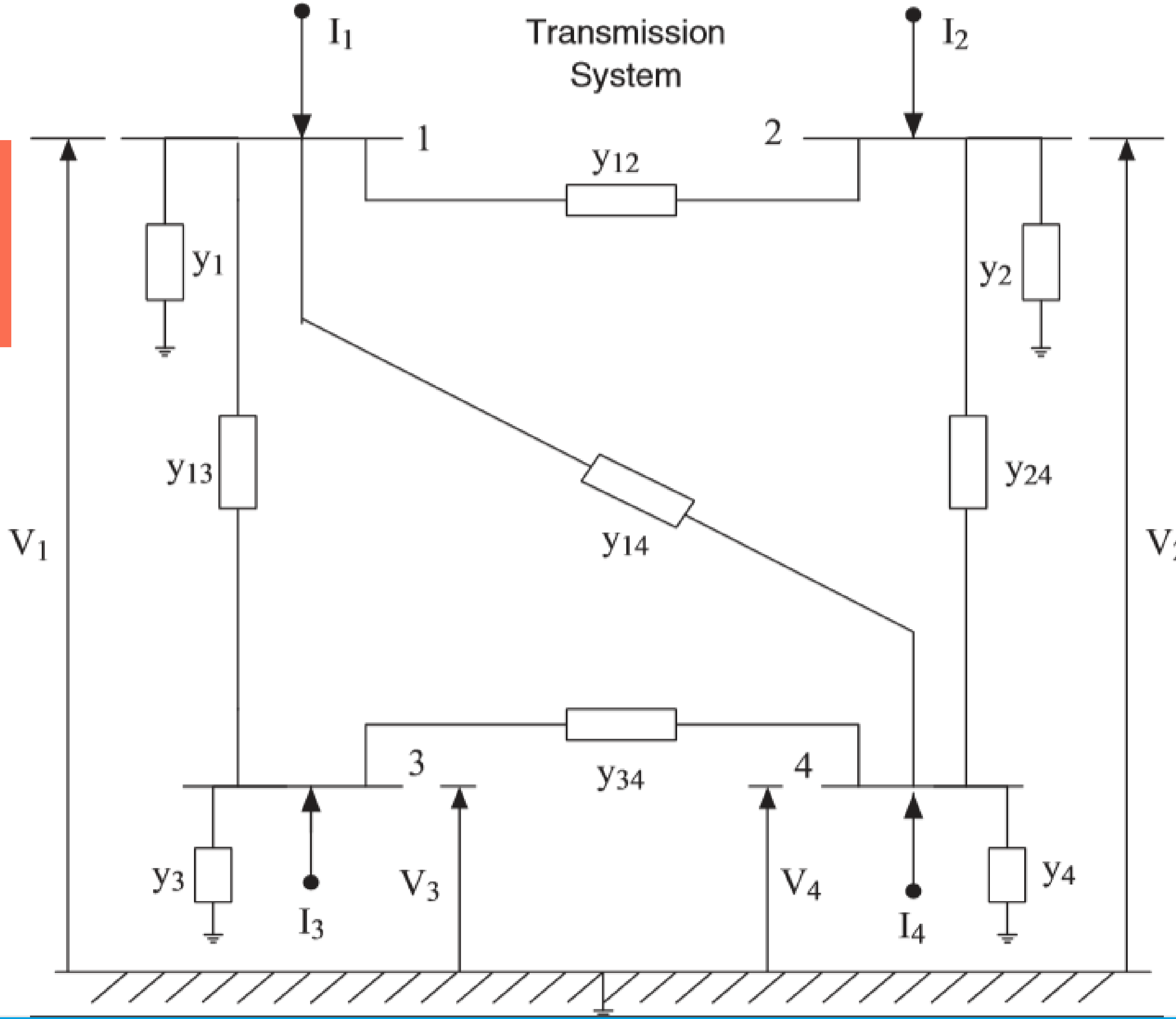
The y'_{ij} is the half of the shunt admittance between bus i and j.

- Representing the series primitive impedance at the lines by their corresponding primitive admittance form where:

$$y_{12} = 1/Z_{1-2};\ y_{13} = 1/Z_{1-3};\ y_{14} = 1/Z_{1-4};\ y_{24} = 1/Z_{2-4};\ y_{34} = 1/Z_{3-4}$$

the power system shown before can be redrawn as follows:

The Current Injection Model Using the Admittance Representation.



- where:

$y_1 = y'_{12} + y'_{13} + y'_{14}$ Total shunt admittance connected to bus 1

$y_2 = y'_{12} + y'_{24}$ Total shunt admittance connected to bus 2

$y_3 = y'_{13} + y'_{34}$ Total shunt admittance connected to bus 3

$y_4 = y'_{14} + y'_{24} + y'_{34}$ Total shunt admittance connected to bus 4

Assuming the ground bus as the reference bus, Kirchhoff ’ s current law for each bus (node) gives:

$$I_1 = V_1 y_1 + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13} + (V_1 - V_4) y_{14}$$
$$I_2 = V_2 y_2 + (V_2 - V_1)\ y_{12} + (V_2 - V_4) y_{24}$$
$$I_3 = V_3 y_3 + (V_3 - V_1)\ y_{13} + (V_3 - V_4) y_{34}$$
$$I_4 = V_4 y_4 + (V_4 - V_1) y_{14} + (V_4 - V_2) y_{24} + (V_4 - V_3) y_{34}$$

These equations can be written as

$$
\begin{aligned}
I_1 &= V_1(y_1 + y_{12} + y_{13} + y_{14}) + V_2(-y_{12}) + V_3(-y_{13}) + V_4(-y_{14}) \\
I_2 &= V_1(-y_{12}) + V_2(y_2 + y_{12} + y_{24}) + V_3(0) + V_4(-y_{24}) \\
I_3 &= V_1(-y_{13}) + V_2(0) + V_3(y_3 + y_{13} + y_{34}) + V_4(-y_{34}) \\
I_4 &= V_1(-y_{14}) + V_2(-y_{24}) + V_3(-y_{34}) + V_4(y_4 + y_{14} + y_{24} + y_{34})
\end{aligned}
$$

In matrix form

$$
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}
$$

- We can formalize the formulation of a bus admittance matrix. This formulation is known as an “ algorithm. ” An algorithm is used to solve a problem using a finite sequence of steps. In 825 AD, Al-Khwarizmı, a Persian astronomer and mathematician wrote, On Calculation with Hindu Numerals. His work was translated into Latin as Algoritmi de Numero Indorum in the 12th century. The words, algebra and algorithm are derived from Al-Khwarizmı’ s treatise.
- Later, Omar Khayyam (1048–1122) the renowned poet, mathematician, and astronomer wrote Demonstrations of Problems of Algebra (1070), which laid down the principles of algebra. He also developed algorithms for the root extraction of arbitrarily high-degree polynomials. Since 825 AD, the word algorithm has been used by mathematicians to formulate and solve complex problems.

3.7 The bus admittance model

- The elements of Y Bus matrix can be calculated from the following algorithm.

Step1. If $i=j$, $Y_{ii}=\Sigma y$, the Σ of admittances connected to bus i

Step2. If $i \neq j$, and bus i not connected to bus j then the element $Y_{ij}=0$

Step3. If $i \neq j$, and bus i is connected to bus j through the admittance y_{ij} then the element is $Y_{ij}=- y_{ij}$

- In a more compact form, we can express the bus current injection vector into a power grid in terms of a bus admittance matrix and a bus voltage vector:

$$[I_{Bus}]=[Y_{Bus}][V_{Bus}]$$

- The YBus matrix model of the power grid is a symmetric, complex, and sparse matrix. The row sum (or column sum) corresponding to each bus, is equal to the admittance to the reference bus. If there is no connection to a reference bus every row sum is zero. For this case, the YBus matrix is singular and det[Y Bus]=0 , and such a YBus matrix cannot be inverted.

3.8 The bus impedance matrix model

- From the Current Injection Model Using the Admittance Representation model it can be seen that the bus current injections are related to bus voltages by the bus admittance matrix as given below:

$$[I_{Bus}]=[Y_{Bus}][V_{Bus}]$$
$$[V_{Bus}]=[Z_{Bus}][I_{Bus}]$$
$$[Z_{Bus}]=[Y_{Bus}]^{-1}$$

- Therefore, the ZBus matrix is the inverse of the YBus matrix. Now the bus voltage vector is expressed in terms of ZBus , which is the bus impedance matrix and IBus is the bus injected current vector. For the system of the Current Injection Model Using the Admittance Representation, the impedance matrix can be expressed as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

- **Problem 3.**

For the power grid given below, compute the bus admittance and bus impedance models.

The admittance matrix is calculated as

$$Y_{11} = y_1 + y_{12} + y_{14} = \frac{1}{.01} + \frac{1}{.01} + \frac{1}{.01} = 300,\ Y_{12} = -y_{12} = -\frac{1}{.01} = -100,$$

$$Y_{14} = -y_{14} = -\frac{1}{.01} = -100,\ Y_{21} = -y_{21} = -\frac{1}{.01} = -100,$$

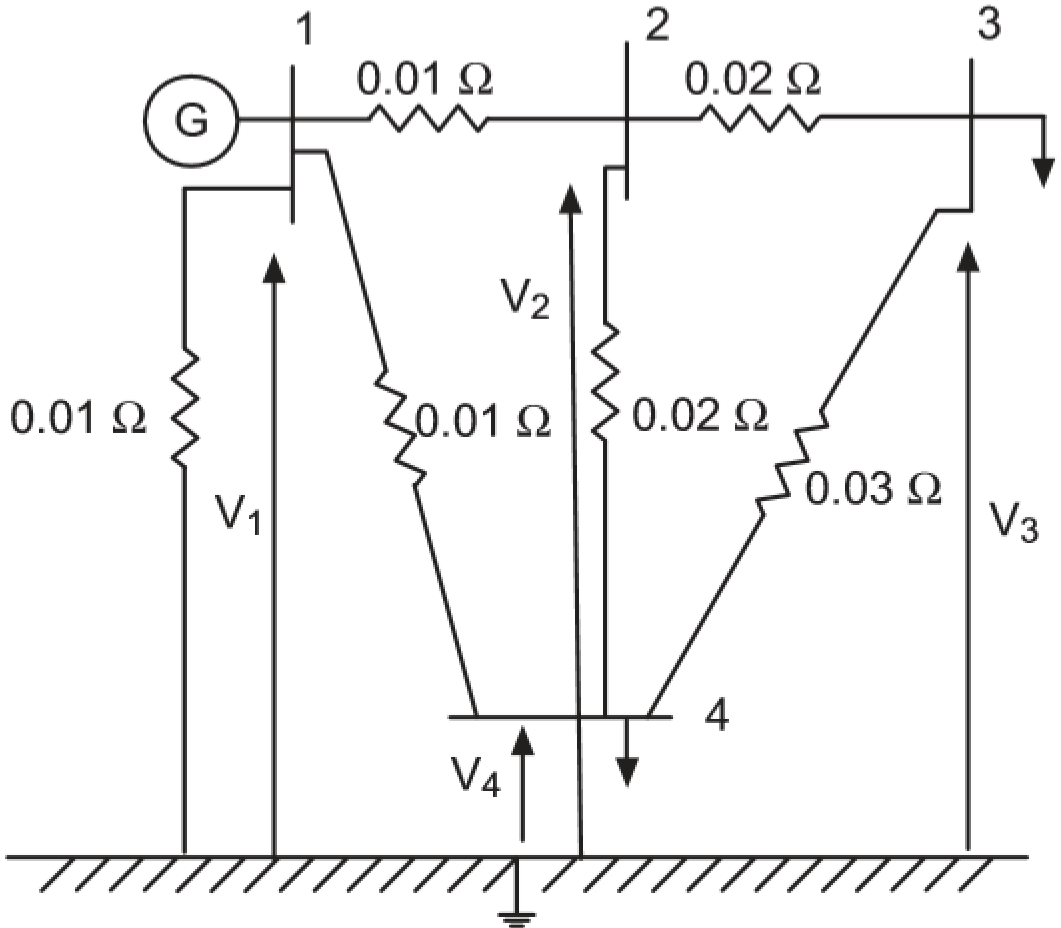
$$Y_{22} = y_{12} + y_{23} + y_{24} = \frac{1}{.01} + \frac{1}{.02} + \frac{1}{.02} = 200,\ Y_{23} = -y_{23} = -\frac{1}{.02} = -50,$$

$$Y_{24} = -y_{24} = -\frac{1}{.02} = -50,\ Y_{32} = -y_{23} = -\frac{1}{.02} = -50,$$

$$Y_{33} = y_{32} + y_{34} = \frac{1}{.02} + \frac{1}{.03} = 83.33,\ Y_{34} = -y_{34} = -\frac{1}{.03} = -33.33,$$

$$Y_{41} = -y_{14} = -\frac{1}{.01} = -100,\ Y_{42} = -y_{24} = -\frac{1}{.02} = -50,\ Y_{43} = -y_{34} = -\frac{1}{.03} = -33.33,$$

$$Y_{44} = y_{41} + y_{42} + y_{43} = \frac{1}{.01} + \frac{1}{.02} + \frac{1}{.03} = 183.33.$$



- **Problem 3.**
- The admittance matrix elements are zero if there are no direct connections between the buses.

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} 300 & -100 & 0 & -100 \\ -100 & 200 & -50 & -50 \\ 0 & -50 & 83.33 & -33.33 \\ -100 & -50 & -33.33 & 183.33 \end{bmatrix}$$

$$Z_{Bus} = Y_{Bus}^{-1} = \begin{bmatrix} 0.010 & 0.010 & 0.010 & 0.010 \\ 0.010 & 0.017 & 0.015 & 0.013 \\ 0.010 & 0.015 & 0.027 & 0.015 \\ 0.010 & 0.013 & 0.015 & 0.017 \end{bmatrix}$$

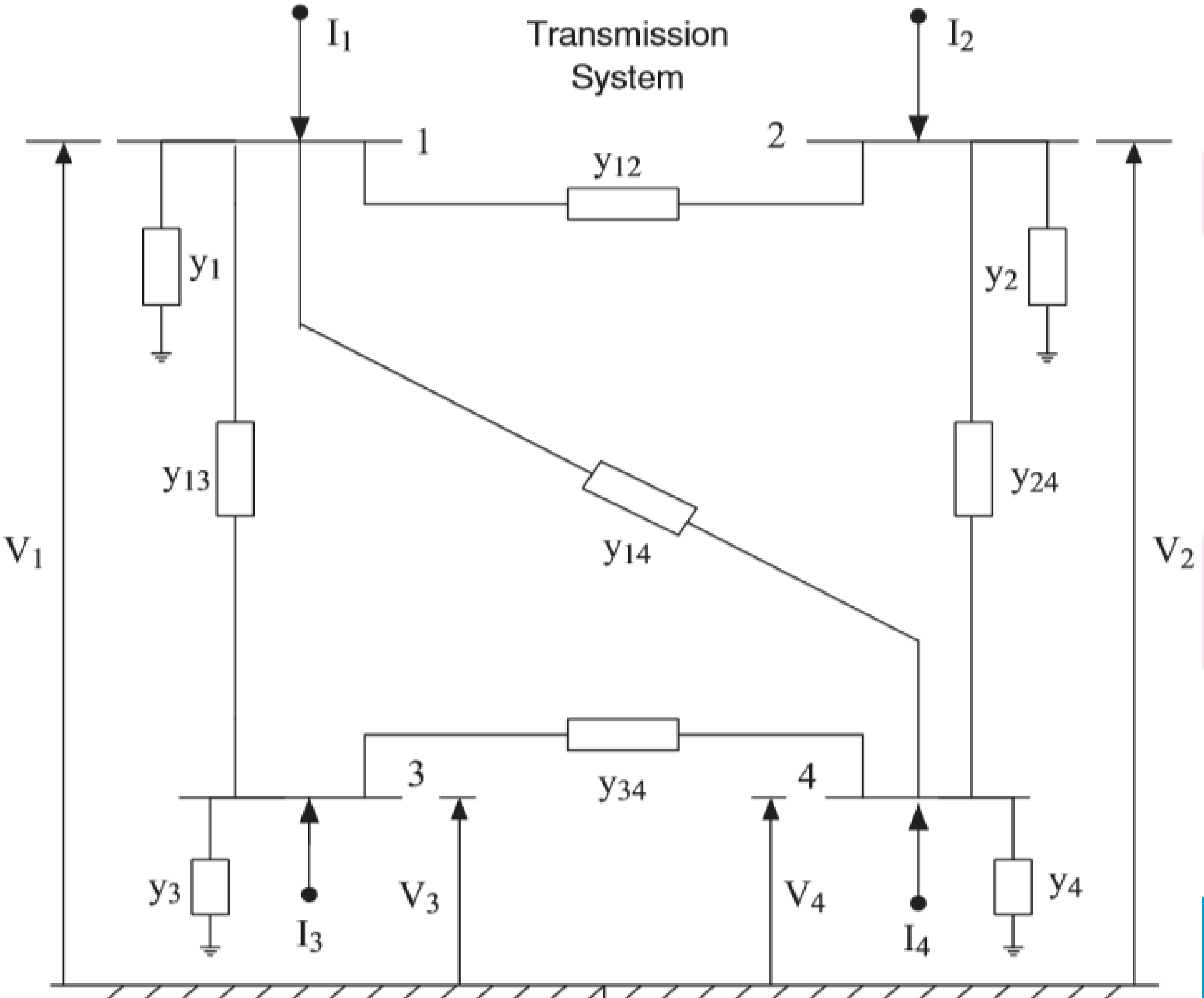
3.9 Formulation of the load flow problem

- Consider again the power grid

The power flow problem can mathematically be stated as given by a bus admittance matrix

$$[I_{Bus}]=[Y_{Bus}][V_{Bus}]$$

The vector of the current injection represents the net injection where the injected current is algebraically **positive** for power **generation** and **negative** for **loads**. Therefore, if the generation at a bus is larger than the load connected to the bus, then there is a positive net injection into the power grid. Otherwise, it will be negative if there are more loads connected to the bus than generating power.



3.9 Formulation of the load flow problem

- Therefore, for each bus k we have

$$S_k = V_k I_k^* \qquad k = 1,2,\ldots..n$$

- Using both eqs we get

$$S_k = V_k \left(\sum_{j=1}^{n} Y_{kj} V_k \right)^*$$

- For each bus k we have a complex equation of the form given by above equation. Therefore, we have n nonlinear complex equations

$$Y_{kj} = G_{kj} + jB_{kj},\ \theta_{kj} = \theta_k - \theta_j$$

$$V_j = V_j \left(\cos\theta_j + j\sin\theta_j\right)$$

$$V_k = V_k \left(\cos\theta_k + j\sin\theta_k\right)$$

3.9 Formulation of the load flow problem

$$I_k^* = \left(\frac{S_k}{V_k}\right)^* = \frac{(P_k + jQ_k)^*}{V_k^*} = \frac{P_k - jQ_k}{V_k \cdot \angle -\theta_k}$$

- where n is the total number of buses in the power grid network.
- From the YBus model, we have the relationship of injected current into the power grid as it relates to the network admittance model, as well as how the power will flow in the transmission system based on the bus voltages. Therefore, for each bus k based on the bus admittance model, we have the following expressions.

$$\frac{P_1 - jQ_1}{V_1^*} = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4$$

$$\frac{P_2 - jQ_2}{V_2^*} = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$

$$\frac{P_3 - jQ_3}{V_3^*} = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4$$

$$\frac{P_4 - jQ_4}{V_4^*} = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4$$

- Rewriting the previous equations as

$$P_1 - jQ_1 = Y_{11}V_1^2 + Y_{12}V_1^*V_2 + Y_{13}V_1^*V_3 + Y_{14}V_1^*V_4$$
$$P_2 - jQ_2 = Y_{21}V_1V_2^* + Y_{22}V_2^2 + Y_{23}V_2^*V_3 + Y_{24}V_2^*V_4$$
$$P_3 - jQ_3 = Y_{31}V_1V_3^* + Y_{32}V_2V_3^* + Y_{33}V_3^2 + Y_{34}V_3^*V_4$$
$$P_4 - jQ_4 = Y_{41}V_1V_4^* + Y_{42}V_2V_4^* + Y_{43}V_3V_4^* + Y_{44}V_{44}^2$$

- The above systems of equations are complex and nonlinear. As we stated, one bus of the system is selected as a **swing bus** and its voltage magnitude is set to **1 p.u;** its **phase angle is set to zero** as the reference phasor. The swing bus will ensure the balance of power between the system loads and the system generations. In a power flow problem, the load bus voltages are the unknown variables and all the injected powers are known variables. Here, there are three nonlinear complex equations to be solved for bus load voltages.

3.9 Formulation of the load flow problem

- In general, as we discussed before, for each bus k a complex equation can be written as two equations in terms of real numbers. Using the above expressions in general formulation, we have

$$P_k = V_k \sum_{j=1}^{n} V_j \left(G_{kj} \cos\theta_{kj} + B_{kj} \sin\theta_{kj}\right)$$

$$Q_k = V_k \sum_{j=1}^{n} V_j \left(G_{kj} \sin\theta_{kj} - B_{kj} \cos\theta_{kj}\right)$$

$$Y_{kj} = G_{kj} + jB_{kj},\ \theta_{kj} = \theta_k - \theta_j$$

where

$$V_k = V_k \left(\cos\theta_k + j\sin\theta_k\right)$$

$$V_j = V_j \left(\cos\theta_j + j\sin\theta_j\right)$$

$$I_k = \left(\frac{S_k}{V_k}\right)^* = \frac{P_k - jQ_k}{V_k^*} = \frac{P_k - jQ_k}{V_k \cdot \angle -\theta_k}$$

- If the system has n buses, the above equations can be expressed as 2n equations:

$$f_1(V_1 \dots\dots\dots V_n, \theta_1 \dots\dots\dots\dots\dots\dots\dots \theta_n) = 0$$
$$f_2(V_1 \dots\dots\dots V_n, \theta_1 \dots\dots\dots\dots\dots\dots\dots \theta_n) = 0$$
$$f_n(V_1 \dots\dots\dots V_n, \theta_1 \dots\dots\dots\dots\dots\dots\dots \theta_n) = 0$$
$$f_{2n}(V_1 \dots\dots\dots V_n, \theta_1 \dots\dots\dots\dots\dots\dots\dots \theta_n) = 0$$

- The above 2n equations can be expressed as:

  where the elements of vector X represent

  the magnitude of voltage and phase angle.

$$F(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ f_{2n}(x) \end{bmatrix}$$

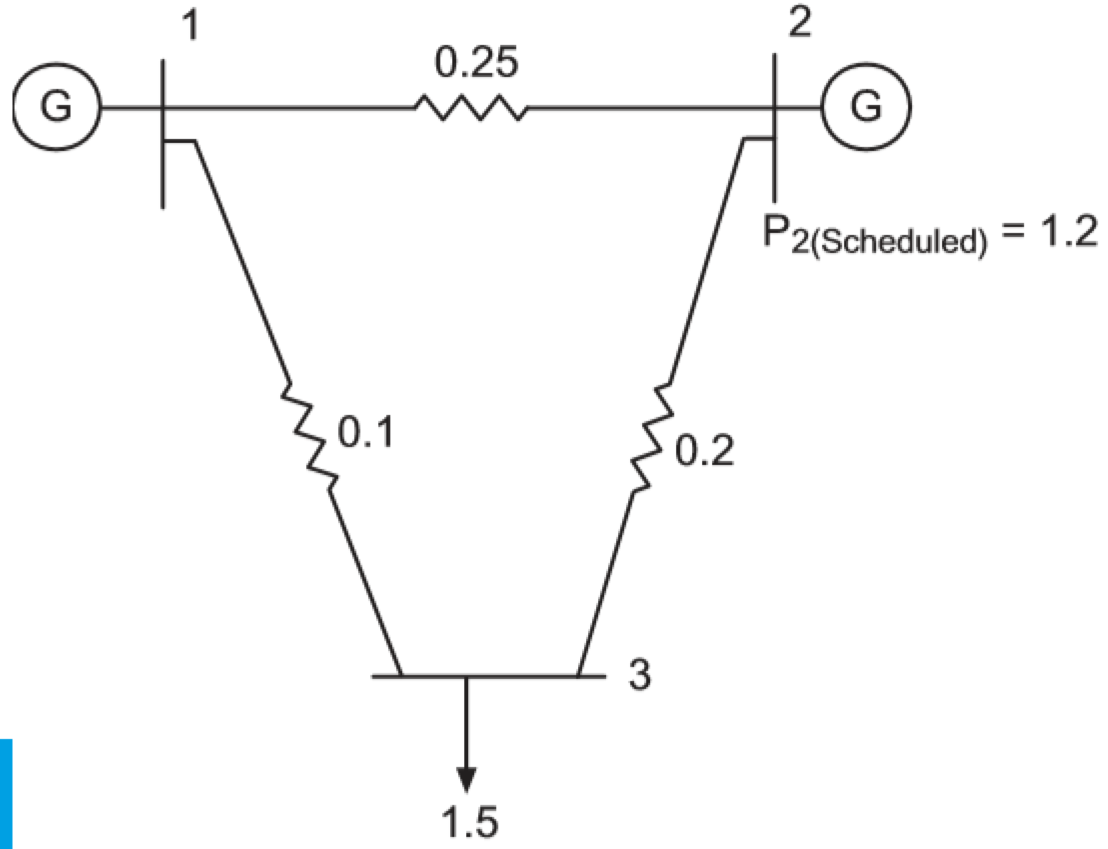
- In this equation, we have 2n nonlinear equations to be solved and vector X can be presented as

$$[X]^t = [V_1, \ldots\ldots\ldots\ldots V_n,\ \theta_1 \ldots\ldots\ldots\ldots\ldots\ldots \theta_n]$$
$$= [x_1, \ldots\ldots\ldots\ldots x_n, x_{n+1} \ldots\ldots\ldots\ldots\ldots\ldots x_{2n}]$$

3.10 Using the Gauss-Seidel algorithm in problems

- For the system below, use the Gauss – Seidel YBus method and solve for the bus voltages.

- bus 1 is the swing bus and its voltage is $V1 = 1 \angle 0$.
- The scheduled power at bus 2 is 1.2 p.u.
- The load at bus 3 is 1.5 p.u.
- Compute the bus 2 and bus 3 voltages.



3.10 Using the Gauss-Seidel algorithm in problems

- To solve this problem, we need to formulate the bus admittance matrix.

$$Y_{Bus} = \begin{bmatrix} 14 & -4 & -10 \\ -4 & 9 & -5 \\ -10 & -5 & 15 \end{bmatrix}$$

- The power flow models are $V_1 = 1\angle 0$

$$I_{Bus} = V_{Bus} \cdot Y_{Bus}$$

- For a DC system, we only have active power flow.

$$I_k^* = I_k, V_k^* = V_k$$

$$S_k = V_k \cdot I_k^* = V_k I_k = P_k, \text{and } Q_k = 0$$

- Therefore, for bus 2 we have $1.2 = V_2 I_2$ and for bus 3: $-1.5 = V_3 I_3$

3.10 Using the Gauss-Seidel algorithm in problems

- We can start the iterative approximation for i = 0 iteration by assuming bus 2 and bus 3 voltages are equal to 1 p.u

$$V_{Bus}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- For bus 2 we have $\sum_{\substack{j=1 \\ j\neq 2}}^{3} Y_{2j}V_j = Y_{21}V_1 + Y_{23}V_3 = (-4)(1) + (-5)(1)$

- Next, we update V2

$$V_2 = \frac{\dfrac{P_2 - jQ_2}{V_2} - \sum_{\substack{j=1 \\ j\neq 2}}^{n} Y_{2j}V_j}{Y_{22}} \qquad i = 1, \ldots\ldots\ldots, j \neq i$$

$$V_2 = \frac{1}{Y_{22}}\left[\frac{1.2}{V_2} - \{(-4)(1.0) + (-5)(1)\}\right]$$

3.10 Using the Gauss-Seidel algorithm in problems

- The updated bus 2 voltage is given as

$$V_2^{(1)} = \frac{1}{9}\left[\frac{1.2}{1} + 4 + 5\right] = 1.1333\ p.u$$

- We continue the iterative process and update bus 3 voltage

$$V_3 = \frac{1}{Y_{33}}\left[\frac{-1.5}{V_3} - [Y_{31}V_1 + Y_{32}V_2]\right]$$

$$V_3 = \frac{1}{15}\left[\frac{-1.5}{1.0} - \{(-10)(1) + (-5)(1.1333)\}\right]$$

$$V_3^{(1)} = \frac{1}{15}[-1.5 + 10 + 5.666] = \frac{14.1666}{15} = 0.9444\ p.u$$

3.10 Using the Gauss-Seidel algorithm in problems

- We continue the approximation by calculating the mismatch at bus 2 and bus 3. The mismatch at bus 2 is

$$P_{2(Calculated)} = \sum_{\substack{j=0 \\ j\neq 2}}^{3} V_2 I_{2j}$$

$$P_{2(Calculated)} = V_2 I_{20} + V_2 I_{21} + V_2 I_{23}$$

$$P_{2(Calculated)} = 0 + 1.1333\left(\frac{V_2 - V_1}{0.25}\right) + 1.333\left(\frac{V_2 - V_3}{0.2}\right)$$

$$P_{2(calculated)} = 0 + 1.1333\left(\frac{1.1333 - 1.0}{0.25}\right) + 1.1333\left(\frac{1.1333 - 0.944}{0.2}\right) = 1.6769$$

$$\Delta P_2 = P_{2(Scheduled)} - P_{2(Calculated)} = 1.2 - 1.6769$$

$$\Delta P_2 = -0.4769\ p.u$$

- The mismatch at bus 3 is

$$P_{3(Calculated)} = \sum_{\substack{j=0 \\ j\neq 3}}^{3} V_3 I_{3j}$$

$$P_{3(Calculated)} = V_3 I_{30} + V_3 I_{31} + V_3 I_{32}$$

$$P_{3(Calculated)} = 0 + 0.944\left(\frac{V_3 - V_1}{0.1}\right) + 0.944\left(\frac{V_3 - V_2}{0.2}\right)$$

$$P_{3(Calculated)} = 0.944\left(\frac{0.944 - 1.0}{0.1}\right) + 0.944\left(\frac{0.944 - 1.1333}{0.2}\right) = -1.4221$$

$$\Delta P_3 = P_{3(Scheduled)} - P_{3(Calculated)} = -1.5 - (-1.4221)$$

$$\Delta P_3 = -0.0779\ p.u$$

3.10 Using the Gauss-Seidel algorithm in problems

- The process is continued until the error reduces to a satisfactory value. The result is obtained after seven iterations. The results are given in Table below. The power supplied by bus 1, the swing bus is equal to the total load minus total generation by all other buses plus the losses.

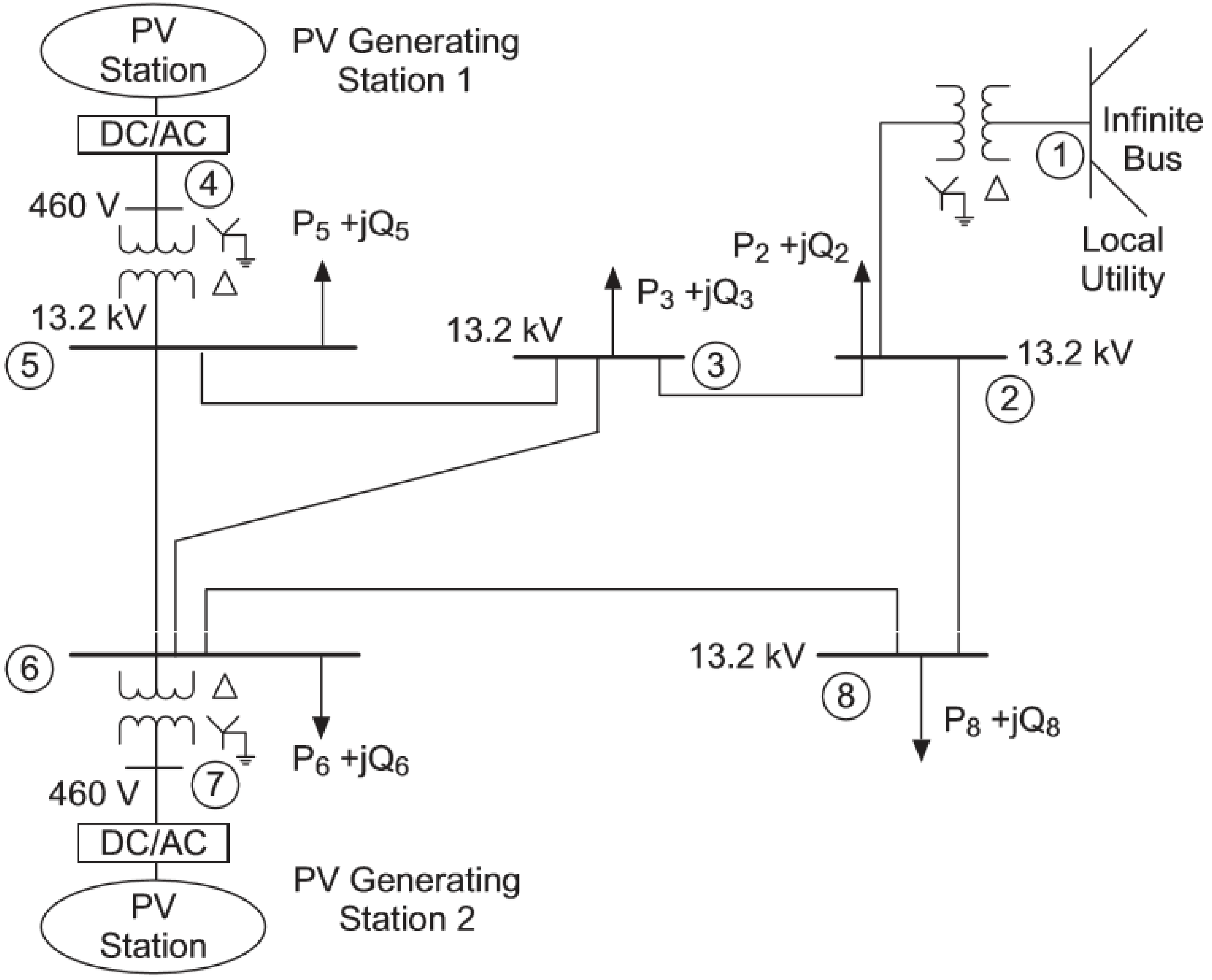
Bus	p.u Voltage	p.u Power Mismatch
2	1.078	0.63×10^{-4}
3	0.917	0.28×10^{-4}

$$P_1 = V_1 I_1 = V_1 \sum_{j=1}^{3} Y_{1j} V_j = V_1 (Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3)$$

- The bus voltage of swing bus is $1\angle 0$ and the p.u power injected by bus is P1=0.522 p.u. The total power loss of the transmission lines is 0.223 p.u.

3.11 The synchronous and asynchronous operation of microgrids

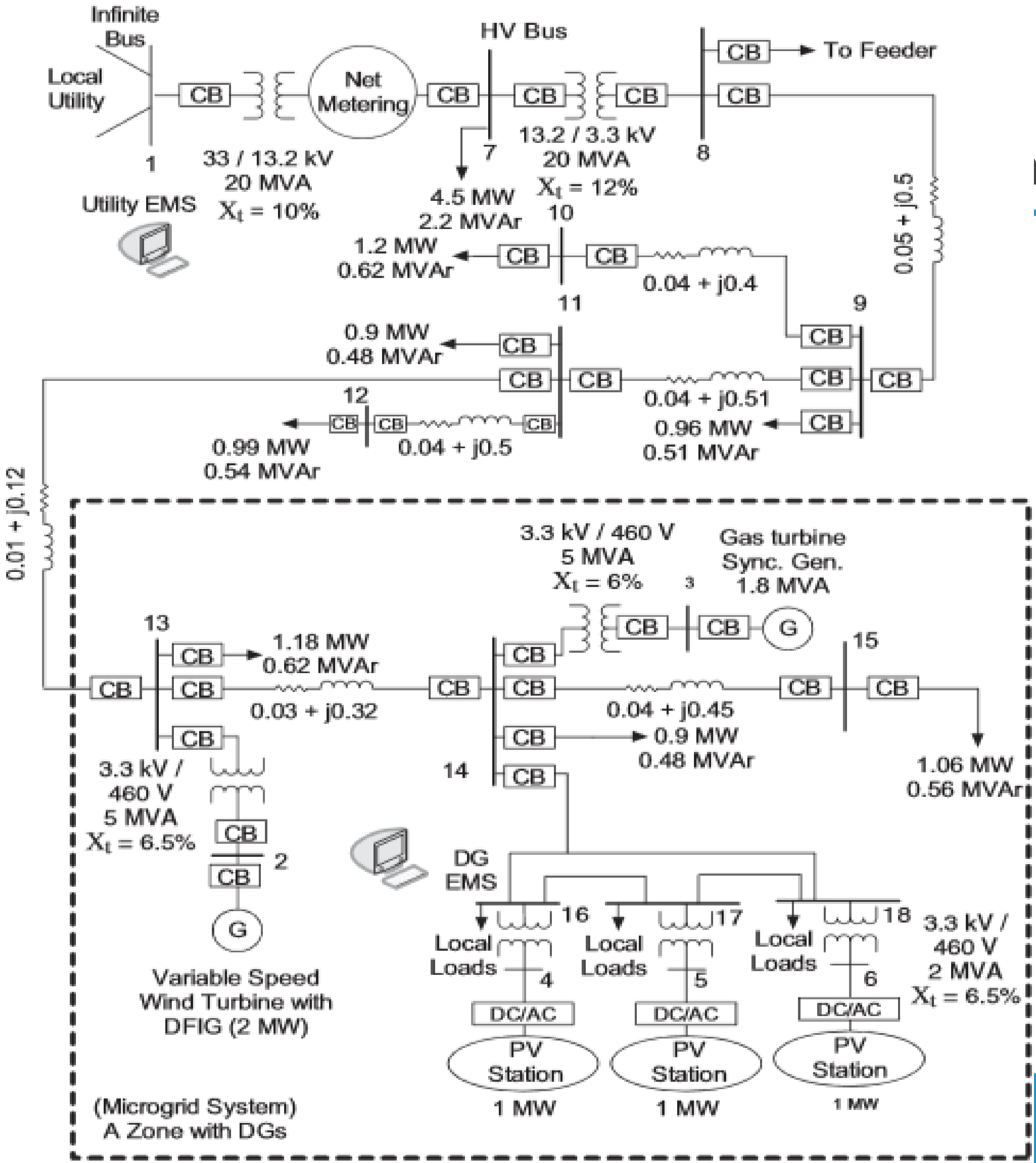
- Figure depicts a typical microgrid connected to a local power grid.
- Depending on the size of the microgrid generation sources, the local network can be designed to operate in a 480 V to 20 kV voltage class.



3.11 The synchronous and asynchronous operation of microgrids

- It is clear that in this microgrid, bus 1 must be designated as a swing bus because the amount of power available at the local power grid is many times larger than the PV microgrid system.
- The operation of the microgrid of the previous Fig. can take two forms: (1) The microgrid can operate as part of the interconnected system, and (2) the microgrid can operate as a standalone once it is separated from the local network. When the microgrid is operating as part of the local power grid, the load and frequency control and voltage control are the responsibility of the local power - grid control center.
- When a microgrid of PV or wind is connected to the local power grid, the entire system operates at a single frequency. The voltage of the power grid bus is also controlled by the local control center. However, the microgrid load buses can still have low voltages if adequate VAr support is not provided. However, the microgrid of PV or wind can be designed to operate asynchronously. Figure next slide shows this.

ısynchronous operation of microgrids

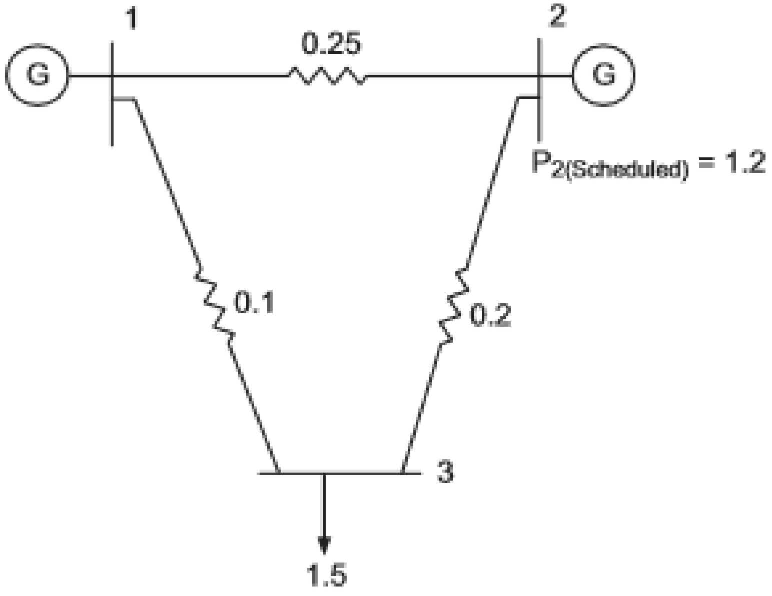


Synchronous and Asynchronous Operation of a Microgrid

3.11 The synchronous and asynchronous operation of microgrids

- The microgrid distributed generation system of this Fig. is designed to operate both as a synchronous and asynchronous system. This distributed generation system has a variable - speed wind doubly - fed induction generator (DFIG) and a gas turbine synchronous generator. When this microgrid is operating as part of the local power grid, the frequency control and voltage control are the responsibility of the local power- grid control center.
- The system operator also controls the local power-grid bus voltages. The gas turbine generator can be operated as a P - V bus: the bus voltage and active power injected into the microgrid are fixed and the reactive power and phase angle are computed in a load fl ow analysis. The DFIG generating station is modeled as a P-V bus that is injecting only active power at a constant voltage.
- When the distributed generation section of a microgrid is separated from the local power grid, the gas turbine unit is responsible for both voltage control and load frequency control. In this case, the gas turbine unit should be modeled as a swing bus for power flow analysis. To ensure stable operation, the local load control of the distributed generation system is essential. If the load control is also provided, this independent microgrid is termed a smart microgrid because it can remain stable with control over its loads.

- **Problem 4:** Consider the three - bus power system given in Fig. below. The system data are as follows:



a. Bus 1 is a swing bus with V1 = 1∠ 0.
b. Scheduled injected power at bus 2 is 1.2 p.u.
c. S cheduled load (negative injection) at bus 3 is 1.5 p.u.

The bus admittance matrix of the system is

$$Y_{Bus} = \begin{bmatrix} 14 & -4 & -10 \\ -4 & 9 & -5 \\ -10 & -5 & 15 \end{bmatrix}$$

3.12 A Newton-Raphson paradigm

- The bus powers can be expressed as nonlinear functions of bus voltages in residue form as

$$P_1(V_1, V_2, V_3) - V_1(Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3) = 0$$

$$P_2(V_1, V_2, V_3) - V_2(Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3) = 0$$

$$P_3(V_1, V_2, V_3) - V_3(Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3) = 0$$

- Using a Taylor series expansion about a guess solution, i.e., $V_2^{(0)}$ $V_3^{(0)}$ and $V_1^{(0)} = 1$

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \end{bmatrix} = -\begin{bmatrix} \frac{\partial P_1}{\partial V_1} & \frac{\partial P_1}{\partial V_2} & \frac{\partial P_1}{\partial V_3} \\ \frac{\partial P_2}{\partial V_1} & \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial V_1} & \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_3} \end{bmatrix}_{\Big|V^{(0)}} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

- In compact notation, we have

$$[\Delta V] = -[J]^{-1} \times [\Delta P]$$

where

$$P_{2(Calculated)} = V_2^{(0)}(Y_{21}V_1^{(0)} + Y_{22}V_2^{(0)} + Y_{23}V_3^{(0)})$$

$$\Delta P_2 = P_{2(Scheduled)} - P_{2(Calculated)} = 0$$

$$P_{3(Calculated)} = V_3^{(0)}(Y_{31}V_1^{(0)} + Y_{32}V_2^{(0)} + Y_{33}V_3^{(0)})$$

$$\Delta P_3 = P_{3(Scheduled)} - P_{3(Calculated)} = 0$$

since V1 = 1∠0 (slack bus). Therefore ΔV1=0.0. Hence, only bus 2 and bus 3 voltages are to be calculated. The Jacobian matrix is a 2×2 matrix as shown below.

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_3} \end{bmatrix}_{V^{(0)}} \begin{bmatrix} \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

3.12 A Newton-Raphson paradigm

- From the above equation, we can calculate ΔV2 and ΔV3.

$$\begin{bmatrix}\Delta V_2 \\ \Delta V_3\end{bmatrix} = -[J]^{-1}_{|V^{(0)}} \begin{bmatrix}\Delta P_2 \\ \Delta P_3\end{bmatrix}$$

- The above solution can be restated as:

$$\Delta V_2 = V_{2(\text{new})} - V_{2(\text{old})}$$

$$\Delta V_3 = V_{3(\text{new})} - V_{3(\text{old})}$$

$$\begin{bmatrix}V_2 \\ V_3\end{bmatrix}_{|new} = \begin{bmatrix}V_2 \\ V_3\end{bmatrix}_{|old} + [J]^{-1}_{|V^{(0)}} \begin{bmatrix}\Delta P_2 \\ \Delta P_3\end{bmatrix}$$

- The elements of the Jacobian matrix are

$$\frac{\partial P_2}{\partial V_2} = Y_{21}V_1 + 2Y_{22}V_2 + Y_{23}V_3$$

$$\frac{\partial P_2}{\partial V_3} = Y_{23}V_2$$

$$\frac{\partial P_3}{\partial V_2} = Y_{32}V_3$$

$$\frac{\partial P_3}{\partial V_3} = Y_{31}V_1 + Y_{32}V_2 + 2Y_{33}V_3$$

Assuming that $V_2^{(0)}=1$ $V_3^{(0)}=1$, the elements of the Jacobian matrix are

$$\frac{\partial P_2}{\partial V_2} = -4 + 2\times 9 - 5 = 9 \qquad \frac{\partial P_3}{\partial V_2} = -5$$

$$\frac{\partial P_2}{\partial V_3} = -5 \qquad \frac{\partial P_3}{\partial V_3} = -10 - 5 + 2\times 15 = 15$$

- Therefore, the Jacobian matrix is given as $[J]=\begin{bmatrix} 9 & -5 \\ -5 & 15 \end{bmatrix}$
- With $[J]^{-1}=\dfrac{1}{110}\begin{bmatrix} 15 & 5 \\ 5 & 9 \end{bmatrix}$
- The mismatch power at each bus k is

$$P_{2(Calculated)}=V_2^{(0)}(Y_{21}V_1^{(0)}+Y_{22}V_2^{(0)}+Y_{23}V_3^{(0)})$$

$$P_{2(Calculated)}=1.0(-4.(1)+9.(1)-5.(1))=0$$

$$\Delta P_2=P_{2(Scheduled)}-P_{2(Calculated)}$$

$$\Delta P_2=1.2-0=1.2$$

$$P_{3(Calculated)}=V_3^{(0)}(Y_{31}V_1^{(0)}+Y_{32}V_2^{(0)}+Y_{33}V_3^{(0)})$$

$$P_{3(Calculated)}=1.0(-10.(1)-5.(1)+15.(1))=0$$

$$\Delta P_3=P_{3(Scheduled)}-P_{3(Calculated)}$$

$$\Delta P_3=-1.5-0=-1.5$$

$$\begin{bmatrix} V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}+\frac{1}{110}\begin{bmatrix} 15 & 5 \\ 5 & 9 \end{bmatrix}\begin{bmatrix} 1.2 \\ -1.5 \end{bmatrix}$$

$$\begin{bmatrix} V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 1.095 \\ 0.932 \end{bmatrix}$$

This new value is now used in the next iteration. The iteration is continued until error does not go below the satisfactory level. The MATLAB simulation testbed for solving the above problem is given below

```
%Power Flow: Newton Raphson
clc; clear all;
mis_match=0.0001;
Y_bus=[ 1/.25+1/.1     -1/.25          -1/.1;
        -1/.25      1/.25+1/.2       -1/.2;
        -1/.1          -1/.2       1/.1+1/.2];

P_sch = [1; 1.2; -1.5];

N = 3; % no. of buses

% allocate storage for Jacobian
J = zeros(N-1,N-1);

% initial mispatch
V = [1 1 1]';
```

```
P_calc = V.*(Y_bus*V);
mismatch = [P_sch(2:N)-P_calc(2:N)];

iteration=0;
% Newton-Raphson iteration
while (iteration<10)
 iteration=iteration+1;
 % calculate Jacobian
 for i = 2:N
  for j = 2:N
   if (i == j)
    J(i-1,j-1)=Y_bus(i,:)*V+Y_bus(i,i)*V(i);
   else
    J(i-1,j-1)=Y_bus(i,j)*V(i);
   end
  end
 end
 % calculate correction
 correction =inv(J)*mismatch;
```

3.12 A Newton-Raphson paradigm

This new value is now used in the next iteration. The iteration is continued until error does not go below the satisfactory level. The MATLAB simulation testbed for solving the above problem is given below

```
V(2:N) = V(2:N)+correction(1:(N-1));

% calculate mismatch and stop iterating
% if the solution has converged
P_calc = V.*(Y_bus*V);
mismatch = [P_sch(2:N)-P_calc(2:N)];
 if (norm(mismatch,'inf') < mis_match)
  break;
 end
end
iteration
% output solution data
for i = 1:N
 fprintf(1, 'Bus %d:\n', i);
 fprintf(1, ' Voltage = %f p.u\n', abs(V(i)));
 fprintf(1, ' Injected P = %f p.u \n', P_calc(i));
end
```

Results:

- The result is obtained after three iterations for c_p $=1\times10^{-4}$. The power supplied by bus 1, the swing bus is equal to the total load minus total generation by all other bus plus the losses. The bus voltage of swing bus is $1\angle0$ and the p.u power is P1= 0. 522. The total power loss of the transmission lines is 0.223 p.u.

Problem 5

Consider the microgrid given in Fig. next slide.

Assume the following data:

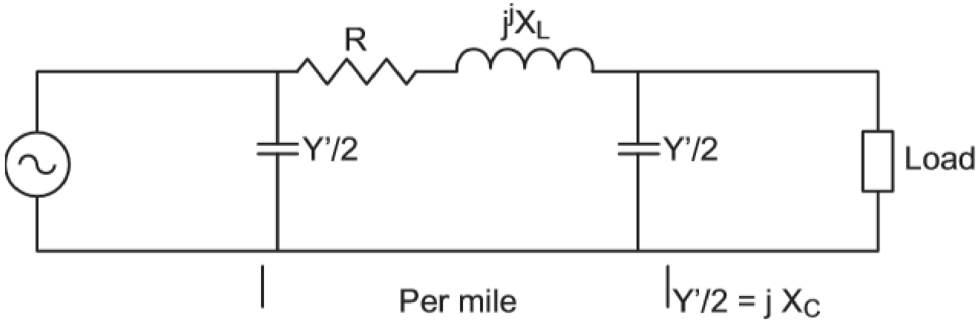
a. Transformers connected to the PV generating station are rated at 460V Y - grounded/13.2 kV Δ, have 10% reactance and 10 MVA capacity. The transformer connected to the power grid is 13.2/63 kV, 10 MVA, 10% reactance.

b. Assume the load on bus 4 is 1.5 MW, 0.9 p.f. lagging, on bus 5 is 2.5 MW, 0.9 p.f. lagging, on bus 6 is 1.0 MW, 0.95 p.f. lagging, on bus 7 is 2 MW, 0.95 p.f. leading, and on bus 8 is 1.0 MW, 0.9 p.f. lagging.

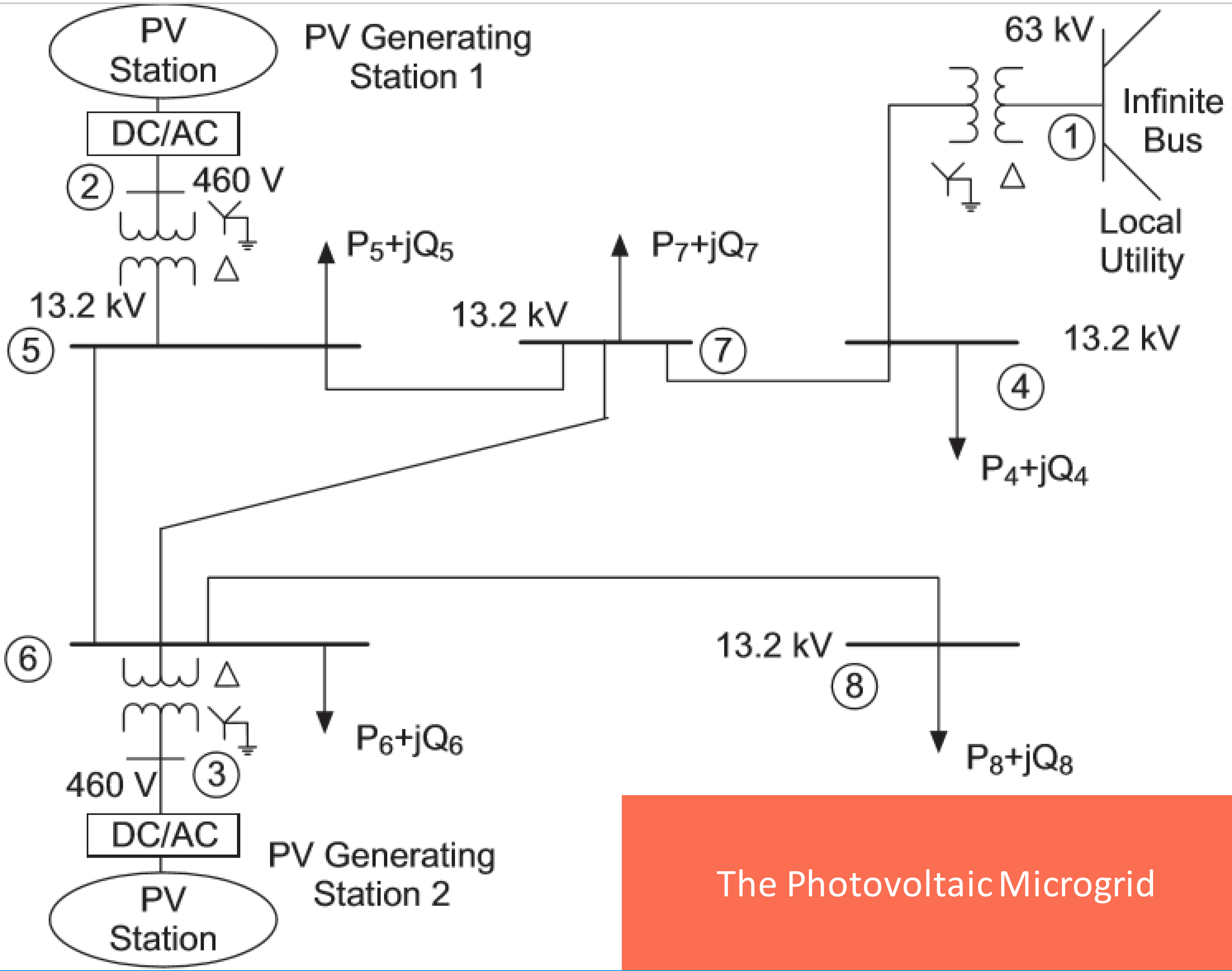
c. The transmission line has a resistance of 0.0685Ω/mile, reactance of 0.40Ω/mile, and half of line charging admittance (Y '/ 2) of 11×10^{-6} Ω^{-1} / mile. The line 4 – 7 is 5 miles, 5 – 6 is 3 miles, 5 – 7 is 2 miles, 6 – 7 is 2 miles, and 6– 8 is 4 miles long.

3.12 A Newton-Raphson paradigm

Problem 5



The Transmission - Line Pie Model



The Photovoltaic Microgrid

d. Assume the PV generating station 1 is rated at 0.75 MW and PV generating station 2 is rated at 3 MW. Assume PV generating stations are operating at unity power factors.

- Perform the following:

i) Assume an S_{base} =10 MVA and a voltage base of 460 V in PV generator #1 and compute the p.u model.

ii) Compute the Y bus model

iii) Compute the load bus voltages using the Newtown–Raphson and Gauss – Seidel methods

iv) Compute the bus voltages and power flow of the microgrid.

v) How much green power is imported or exported to the local power grid?

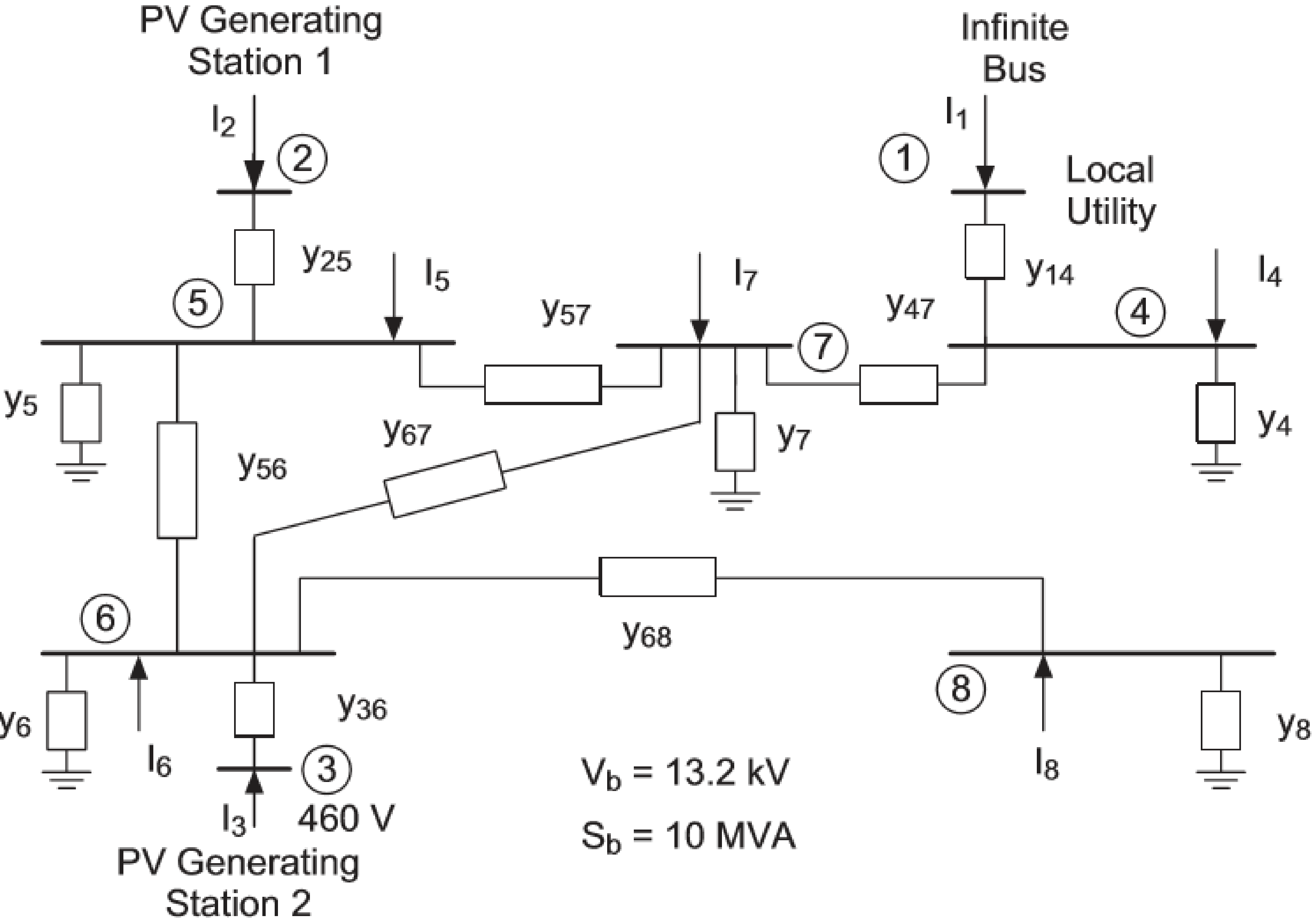
- The base value of the volt-amp is designated as Sb =10 MVA; the voltage base on the PV generator side is specified as 460 V, and the voltage base on the transmission line side is specified as Vb =13 2. kV.
- The base impedance is

$$Z_b = \frac{V_b^2}{S_b} = \frac{\left(13.2\times 10^3\right)^2}{10\times 10^6} = 17.424\ \Omega$$

- The base admittance is given by
-

$$Y_b = \frac{1}{Z_b} = \frac{1}{17.424} = 0.057$$

- The current injection model of the PV system is shown in Fig. next slide.



The Current Injection Model

- From transmission line data, the primitive impedance and admittance are calculated:

$$Z_{1-4\ \mathrm{p.u}} = Z_{2-5\ \mathrm{p.u}} = Z_{3-6\ \mathrm{p.u}} = j0.1,$$

$$y_{14,p.u} = y_{25,p.u} = y_{36,p.u} = \frac{1}{j0.1} = -j10$$

$$Z_{4-7} = 5(0.0685 + j0.4)/Z_b = 0.020 + j0.115,$$

$$y_{47,p.u} = \frac{1}{0.020 + j0.115} = 1.45 - j8.46\ \mathrm{p.u}\ \Omega$$

$$Z_{5-6} = 3(0.0685 + j0.4)/Z_b = 0.012 + j0.069,$$

$$y_{56,p.u} = \frac{1}{0.012 + j0.069} = 2.42 - j14.11\ \mathrm{p.u}\ \Omega$$

$$Z_{5-7} = Z_{6-7} = 2(0.0685 + j0.4)/Z_b = 0.008 + j0.046\ \mathrm{p.u}\ \Omega$$

$$y_{57.} = y_{67,p.u} = \frac{1}{0.008 + j0.046} = 3.62 - j21.16$$

$$Z_{6-8} = 4(0.0685 + j0.4)/Z_b = 0.016 + j0.092,\ \mathrm{p.u}\ \Omega$$

$$y_{68,.} = \frac{1}{0.016 + j0.092} = 1.18 - 10.57$$

- The line - charging admittance is the same as the Y'/2 as shown in the transmission line pie model. The line- charging admittances in per unit are as follows:

$$y'_{47.} = 5\times\frac{j11\times10^{-6}}{Y_b} = j9.58\times10^{-4},\ y'_{56,p.u} = 3\times\frac{j11\times10^{-6}}{Y_b} = j5.75\times10^{-4},$$

$$y'_{57.} = y'_{67,} = 2\times\frac{j11\times10^{-6}}{Y_b} = j3.83\times10^{-4},\ y'_{68,.} = 4\times\frac{j11\times10^{-6}}{Y_b} = j7.67\times10^{-4}$$

- The p.u admittance matrix, YBus is calculated

$$Y_{Bus} = \begin{bmatrix} -j10 & 0 & 0 & j10 & 0 & 0 & 0 & 0 \\ 0 & -j10 & 0 & 0 & j10 & 0 & 0 & 0 \\ 0 & 0 & -j10 & 0 & 0 & j10 & 0 & 0 \\ j10 & 0 & 0 & 1.45-j18.46 & 0 & 0 & -1.45+j8.46 & 0 \\ 0 & j10 & 0 & 0 & 6.04-j45.26 & -2.42+j14.11 & -3.62+j21.16 & 0 \\ 0 & 0 & j10 & 0 & -2.42+j14.11 & 7.85-j55.84 & -3.62+j21.16 & -1.81+j10.58 \\ 0 & 0 & 0 & -1.45+j8.46 & -3.62+j21.16 & -3.62+j21.16 & 8.69-j50.78 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.81+j10.58 & 0 & 1.81-j10.58 \end{bmatrix}$$

- A MATLAB program was written to perform the load flow analysis. The PV generator buses are treated as PV buses in the load flow problem. The voltage at the PV generator buses are fixed at 1 p.u. Table below provides the scheduled powers of each bus.

Bus	2	3	4	5	6	7	8
$P_{scheduled}$	0.075	0.3	–0.150	–0.250	–0.100	–0.200	–0.100
$Q_{scheduled}$	—	—	–0.073	–0.121	–0.033	0.066	–0.048

- For this load flow problem, the tolerance of error chosen was 10^{-5}. The load flow problem was solved using the Newton – Raphson and Gauss – Seidel methods. Next Table gives the results using the Newton – Raphson method, which converges in three iterations.

3.12 A Newton-Raphson paradigm

$S_b = 10\ \text{MVA},\ V_b = 13.2\ \text{kV}$

Bus #	Volts (p.u)	Angle (°)	Generation MW (p.u)	Generation MVAr (p.u)	Load MW (p.u)	Load MVAr (p.u)	ΔP	ΔQ
1	1.000	0.0	0.427	0.065	0	0	0	0
2	1.000	–4.1	0.075	0.094	0	0	0.005×10^{-10}	0
3	1.000	–2.5	0.300	0.085	0	0	0.021×10^{-10}	0
4	0.994	–2.5	0	0	0.150	0.073	0.189×10^{-10}	0.424×10^{-10}
5	0.991	–4.6	0	0	0.250	0.121	0.127×10^{-10}	0.002×10^{-10}
6	0.992	–4.3	0	0	0.100	0.033	0.079×10^{-10}	0.071×10^{-10}
7	0.992	–4.3	0	0	0.200	–0.066	0.233×10^{-10}	0.110×10^{-10}
8	0.986	–4.8	0	0	0.100	0.048	0.074×10^{-10}	0.009×10^{-10}

The Voltages and Power of Each Bus Using the Newton–Raphson Method.

- To maintain PV bus 2 and PV bus 3 at 1 p.u, the required reactive power at bus 2 is 0.094 p.u. and at bus 3 is 0.085 p.u. The active power flow from the local power grid is 0.427 p.u. The reactive power flow from the local power grid is 0.065 p.u.

P_{loss} (p.u) = 0.002, Q_{loss} (p.u) = 0.035			
From Bus #	To Bus #	MW Flow (p.u)	MVAr Flow (p.u)
1	4	0.427	0.065
2	5	0.075	0.094
3	6	0.300	0.085
4	7	0.277	–0.025
5	6	–0.079	–0.006
5	7	–0.097	–0.021
6	7	0.021	–0.012
6	8	0.100	0.049

The Power Flow through Transmission Lines and Transformers Using the Newton– Raphson Method

- **Problem 6**

A three - phase generator rated 440 V, 20 kVA is connected by one cable with impedance of 1+ j0.012Ω to a motor load rated 440 V, 15 kVA, 0.9 p.f. lagging. Assume the load voltage to be set at 5% above its rated value. Perform the following: i) Give the three- phase circuit if the load is Y connected

ii) Give the three- phase circuit if the load Δ connected

iii) Give a one - line diagram

iv) Compute the generator voltage

- **Problem 7**
- A three- phase generator rated 440 V, 20 kVA is connected through one cable with impedance of $1 + j.012\ \Omega$ to a Δ -connected motor load rated 440 V, 10 kVA, 0.9 p.f. lagging. Assume the generator voltage is to be controlled at its rated voltage and its phase angle is used as the reference angle. Perform the following:

i) What is the number of unknown variables?

ii) How many equations are needed to solve for bus voltage? Give the expressions.

iii) Compute the load bus voltage

- **Problem 8.**

The radial feeder of Fig. below is connected to a local power grid rated at 11.3 kV distribution. Assume the base voltage of 10 kVA and a voltage base of 11.3 kV.

Perform the following:

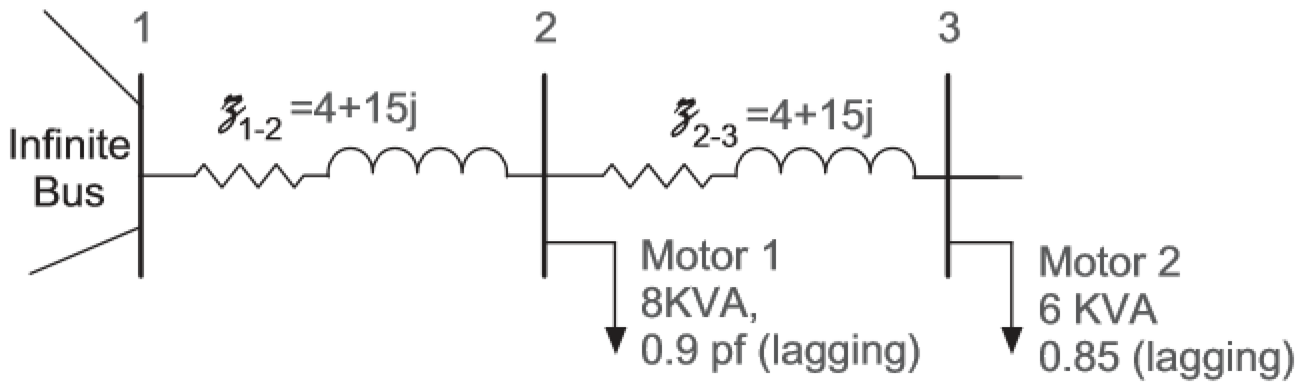
i) Compute the per unit model

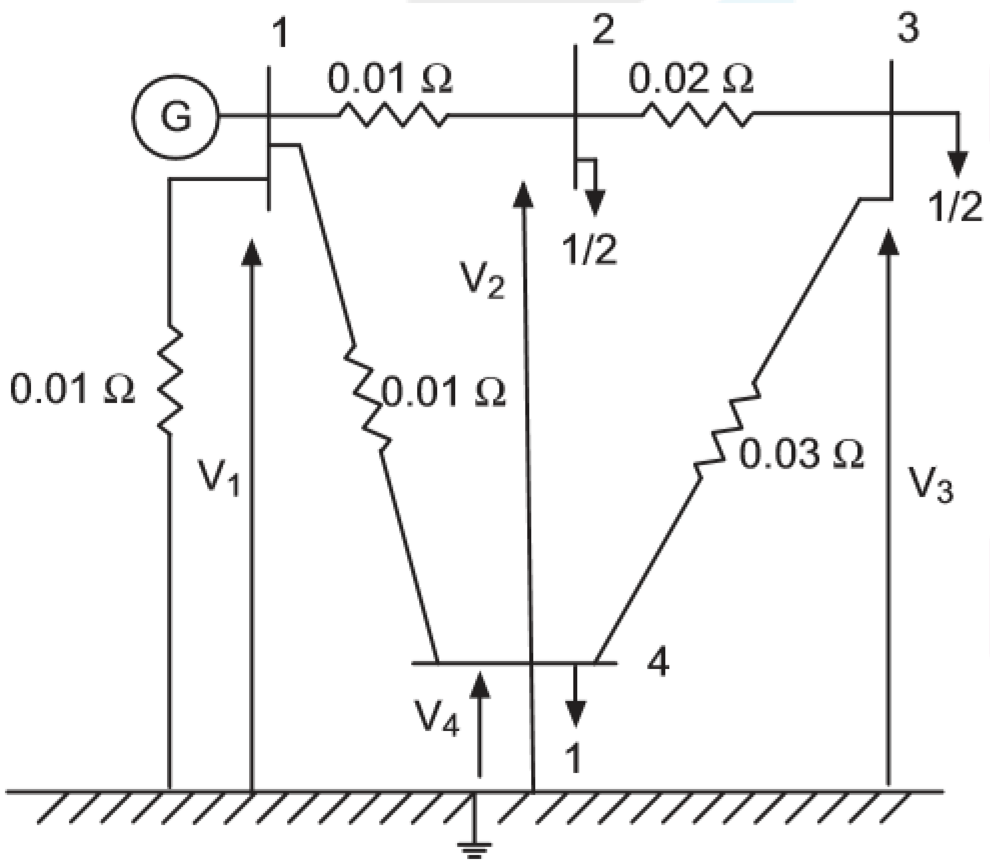
ii) Write the number of equations that are needed to solve for the bus load voltages

iii) Use the Gauss– Seidel method and compute the bus voltages

iv) Compute the power at bus 1. Assume the power mismatch of 0.00001 per unit.

v) Compute the total active and reactive power losses


1
2
3
Infinite Bus
z_{1-2} =4+15j
z_{2-3} =4+15j
Motor 1
8KVA,
0.9 pf (lagging)
Motor 2
6 KVA
0.85 (lagging)


G
1
2
3
0.01 Ω
0.02 Ω
1/2
1/2
0.01 Ω
0.01 Ω
0.03 Ω
V_1
V_2
V_3
V_4
4
1

References

[1] M.H. Mickle and T.W. Sze. Optimization in Systems Engineering. Intext Educational Publishers, 1972.

[2] M.E. El -Hawary. Introduction to Electrical Power Systems Design and Analysis. IEEE Press Series on Power Engineering, 2008.

[3] J.A. Momoh. Electric Power System Application of Optimization (2nd edition). CRC press, 2017.

[4] Stott, Brian, and J. L. Marinho. "Linear programming for power-system network security applications." IEEE Transactions on Power Apparatus and Systems 3 (1979): 837-848., doi: 10.1109/TPAS.1979.319296